CHAPTER

7

Pressure Vessels

7.1 Introduction

The pressure vessels (*i.e.* cylinders or tanks) are used to store fluids under pressure. The fluid being stored may undergo a change of state inside the pressure vessel as in case of steam boilers or it may combine with other reagents as in a chemical plant. The pressure vessels are designed with great care because rupture of a pressure vessel means an explosion which may cause loss of life and property. The material of pressure vessels may be brittle such as cast iron, or ductile such as mild steel.

7.2 Classification of Pressure Vessels

The pressure vessels may be classified as follows:

1. *According to the dimensions.* The pressure vessels, according to their dimensions, may be classified as ***thin shell*** or ***thick shell.*** If the wall thickness of the shell (t) is less than 1/10 of the diameter of the shell (d), then it is called a ***thin shell.*** On the other hand, if the wall thickness of the shell is greater than 1/10 of the diameter of the shell, then it is said to be a ***thick shell.*** Thin shells are used in boilers, tanks and pipes, whereas thick shells are used in high pressure cylinders, tanks, gun barrels etc.

Pressure vessels.

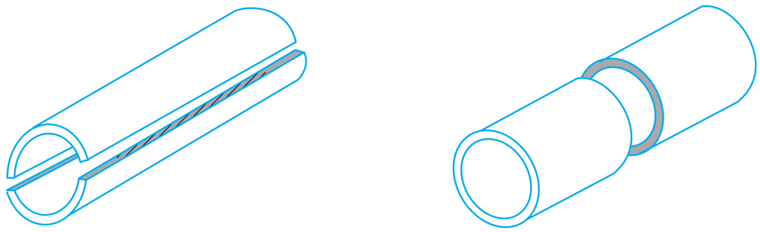
Note: Another criterion to classify the pressure vessels as thin shell or thick shell is the internal fluid pressure (p) and the allowable stress (σ_t). If the internal fluid pressure (p) is less than 1/6 of the allowable stress, then it is called a ***thin shell.*** On the other hand, if the internal fluid pressure is greater than 1/6 of the allowable stress, then it is said to be a ***thick shell.***

2. According to the end construction. The pressure vessels, according to the end construction, may be classified as ***open end*** or ***closed end.*** A simple cylinder with a piston, such as cylinder of a press is an example of an open end vessel, whereas a tank is an example of a closed end vessel. In case of vessels having open ends, the circumferential or hoop stresses are induced by the fluid pressure, whereas in case of closed ends, longitudinal stresses in addition to circumferential stresses are induced.

7.3 Stresses in a Thin Cylindrical Shell due to an Internal Pressure

The analysis of stresses induced in a thin cylindrical shell are made on the following assumptions:

1. The effect of curvature of the cylinder wall is neglected.
2. The tensile stresses are uniformly distributed over the section of the walls.
3. The effect of the restraining action of the heads at the end of the pressure vessel is neglected.



(*a*) Failure of a cylindrical shell along the longitudinal section.

(*b*) Failure of a cylindrical shell along the transverse section.

Fig. 7.1. Failure of a cylindrical shell.

When a thin cylindrical shell is subjected to an internal pressure, it is likely to fail in the following two ways:

1. It may fail along the longitudinal section (*i.e.* circumferentially) splitting the cylinder into two troughs, as shown in Fig. 7.1 (*a*).
2. It may fail across the transverse section (*i.e.* longitudinally) splitting the cylinder into two cylindrical shells, as shown in Fig. 7.1 (*b*).

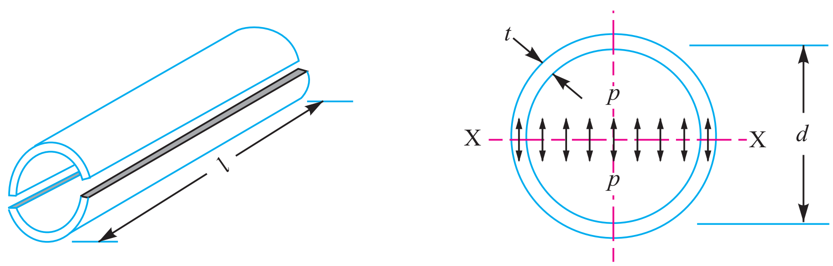
Thus the wall of a cylindrical shell subjected to an internal pressure has to withstand tensile stresses of the following two types:

(*a*) Circumferential or hoop stress, and (*b*) Longitudinal stress.

These stresses are discussed, in detail, in the following articles.

7.4 Circumferential or Hoop Stress

Consider a thin cylindrical shell subjected to an internal pressure as shown in Fig. 7.2 (*a*) and (*b*). A tensile stress acting in a direction tangential to the circumference is called ***circumferential*** or ***hoop stress.*** In other words, it is a tensile stress on *longitudinal section (or on the cylindrical walls).



(*a*) View of shell. (*b*) Cross-section of shell.

Fig. 7.2. Circumferential or hoop stress.

Let p = Intensity of internal pressure,

d = Internal diameter of the cylindrical shell,

l = Length of the cylindrical shell,

t = Thickness of the cylindrical shell, and

σ_{t1} = Circumferential or hoop stress for the material of the cylindrical shell.

We know that the total force acting on a longitudinal section (*i.e.* along the diameter *X-X*) of the shell

$$= \text{Intensity of pressure} \times \text{Projected area} = p \times d \times l \qquad \ldots(i)$$

and the total resisting force acting on the cylinder walls

$$= \sigma_{t1} \times 2t \times l \qquad \ldots(\because \text{ of two sections}) \qquad \ldots(ii)$$

From equations (*i*) and (*ii*), we have

$$\sigma_{t1} \times 2t \times l = p \times d \times l \quad \text{or} \quad \sigma_{t1} = \frac{p \times d}{2t} \quad \text{or} \quad t = \frac{p \times d}{2\,\sigma_{t1}} \qquad \ldots(iii)$$

The following points may be noted:

1. In the design of engine cylinders, a value of 6 mm to 12 mm is added in equation (*iii*) to permit reboring after wear has taken place. Therefore

$$t = \frac{p \times d}{2\,\sigma_{t1}} + 6 \text{ to } 12 \text{ mm}$$

2. In constructing large pressure vessels like steam boilers, riveted joints or welded joints are used in joining together the ends of steel plates. In case of riveted joints, the wall thickness of the cylinder,

$$t = \frac{p \times d}{2\sigma_{t1} \times \eta_l}$$

where η_l = Efficiency of the longitudinal riveted joint.

* A section cut from a cylinder by a plane that contains the axis is called longitudinal section.

3. In case of cylinders of ductile material, the value of circumferential stress (σ_{t1}) may be taken 0.8 times the yield point stress (σ_y) and for brittle materials, σ_{t1} may be taken as 0.125 times the ultimate tensile stress (σ_u).

4. In designing steam boilers, the wall thickness calculated by the above equation may be compared with the minimum plate thickness as provided in boiler code as given in the following table.

Table 7.1. Minimum plate thickness for steam boilers.

Boiler diameter	*Minimum plate thickness (t)*
0.9 m or less	6 mm
Above 0.9 m and upto 1.35 m	7.5 mm
Above 1.35 m and upto 1.8 m	9 mm
Over 1.8 m	12 mm

Note: If the calculated value of *t* is less than the code requirement, then the latter should be taken, otherwise the calculated value may be used.

The boiler code also provides that the factor of safety shall be at least 5 and the steel of the plates and rivets shall have as a minimum the following ultimate stresses.

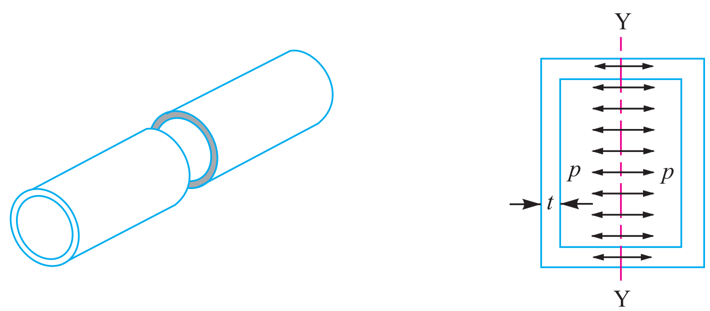
Tensile stress, σ_t = 385 MPa

Compressive stress, σ_c = 665 MPa

Shear stress, τ = 308 MPa

7.5 Longitudinal Stress

Consider a closed thin cylindrical shell subjected to an internal pressure as shown in Fig. 7.3 (*a*) and (*b*). A tensile stress acting in the direction of the axis is called ***longitudinal stress***. In other words, it is a tensile stress acting on the *transverse or circumferential section *Y-Y* (or on the ends of the vessel).



(*a*) View of shell. (*b*) Cross-section of shell.

Fig. 7.3. Longitudinal stress.

Let σ_{t2} = Longitudinal stress.

In this case, the total force acting on the transverse section (*i.e.* along *Y-Y*)

= Intensity of pressure × Cross-sectional area

$$= p \times \frac{\pi}{4}(d)^2 \quad \text{...}(i)$$

and total resisting force $= \sigma_{t2} \times \pi\, d.t$ ...(*ii*)

* A section cut from a cylinder by a plane at right angles to the axis of the cylinder is called transverse section.

Contents

From equations *(i)* and *(ii)*, we have

$$\sigma_{t2} \times \pi\, d.t = p \times \frac{\pi}{4}(d)^2$$

$$\therefore \qquad \sigma_{t2} = \frac{p \times d}{4\,t} \qquad \text{or} \qquad t = \frac{p \times d}{4\,\sigma_{t2}}$$

If η_c is the efficiency of the circumferential joint, then

$$t = \frac{p \times d}{4\sigma_{t2} \times \eta_c}$$

From above we see that the longitudinal stress is half of the circumferential or hoop stress. Therefore, the design of a pressure vessel must be based on the maximum stress *i.e.* hoop stress.

Example 7.1. *A thin cylindrical pressure vessel of 1.2 m diameter generates steam at a pressure of 1.75 N/mm². Find the minimum wall thickness, if (a) the longitudinal stress does not exceed 28 MPa; and (b) the circumferential stress does not exceed 42 MPa.*

Cylinders and tanks are used to store fluids under pressure.

Solution. Given : d = 1.2 m = 1200 mm ; p = 1.75 N/mm² ; σ_{t2} = 28 MPa = 28 N/mm² ; σ_{t1} = 42 MPa = 42 N/mm²

(a) When longitudinal stress (σ_{t2}) does not exceed 28 MPa

We know that minimum wall thickness,

$$t = \frac{p\,.\,d}{4\,\sigma_{t2}} = \frac{1.75 \times 1200}{4 \times 28} = 18.75 \text{ say } 20 \text{ mm} \quad \textbf{Ans.}$$

(b) When circumferential stress (σ_{t1}) does not exceed 42 MPa

We know that minimum wall thickness,

$$t = \frac{p\,.\,d}{2\,\sigma_{t1}} = \frac{1.75 \times 1200}{2 \times 42} = 25 \text{ mm} \quad \textbf{Ans.}$$

Example 7.2. *A thin cylindrical pressure vessel of 500 mm diameter is subjected to an internal pressure of 2 N/mm². If the thickness of the vessel is 20 mm, find the hoop stress, longitudinal stress and the maximum shear stress.*

Solution. Given : d = 500 mm ; p = 2 N/mm² ; t = 20 mm

Top

Contents

Hoop stress

We know that hoop stress,

$$\sigma_{t1} = \frac{p \cdot d}{2\,t} = \frac{2 \times 500}{2 \times 20} = 25 \text{ N/mm}^2 = 25 \text{ MPa } \textbf{Ans.}$$

Longitudinal stress

We know that longitudinal stress,

$$\sigma_{t2} = \frac{p \cdot d}{4\,t} = \frac{2 \times 500}{4 \times 20} = 12.5 \text{ N/mm}^2 = 12.5 \text{ MPa } \textbf{Ans.}$$

Maximum shear stress

We know that according to maximum shear stress theory, the maximum shear stress is one-half the algebraic difference of the maximum and minimum principal stress. Since the maximum principal stress is the hoop stress (σ_{t1}) and minimum principal stress is the longitudinal stress (σ_{t2}), therefore maximum shear stress,

$$\tau_{max} = \frac{\sigma_{t1} - \sigma_{t2}}{2} = \frac{25 - 12.5}{2} = 6.25 \text{ N/mm}^2 = 6.25 \text{ MPa } \textbf{Ans.}$$

Example 7.3. *An hydraulic control for a straight line motion, as shown in Fig. 7.4, utilises a spherical pressure tank 'A' connected to a working cylinder B. The pump maintains a pressure of 3 N/mm² in the tank.*

1. If the diameter of pressure tank is 800 mm, determine its thickness for 100% efficiency of the joint. Assume the allowable tensile stress as 50 MPa.

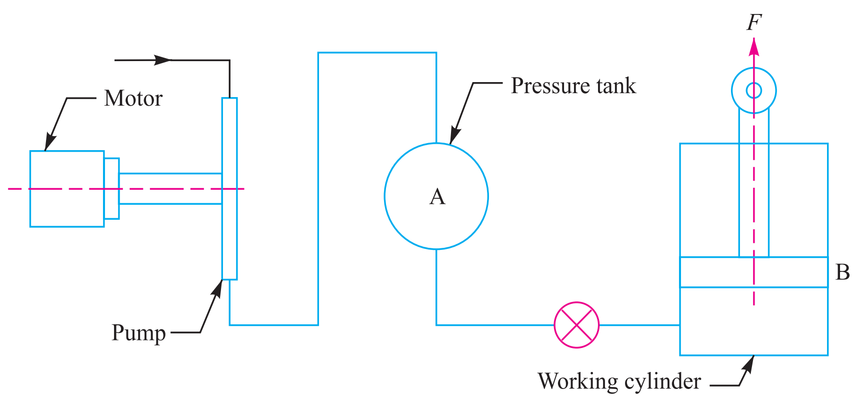


Fig. 7.4

2. Determine the diameter of a cast iron cylinder and its thickness to produce an operating force F = 25 kN. Assume (i) an allowance of 10 per cent of operating force F for friction in the cylinder and packing, and (ii) a pressure drop of 0.2 N/mm² between the tank and cylinder. Take safe stress for cast iron as 30 MPa.

3. Determine the power output of the cylinder, if the stroke of the piston is 450 mm and the time required for the working stroke is 5 seconds.

4. Find the power of the motor, if the working cycle repeats after every 30 seconds and the efficiency of the hydraulic control is 80 percent and that of pump 60 percent.

Solution. Given : $p = 3 \text{ N/mm}^2$; $d = 800$ mm ; $\eta = 100\% = 1$; $\sigma_{t1} = 50 \text{ MPa} = 50 \text{ N/mm}^2$; $F = 25 \text{ kN} = 25 \times 10^3 \text{ N}$; $\sigma_{tc} = 30 \text{ MPa} = 30 \text{ N/mm}^2$: $\eta_H = 80\% = 0.8$; $\eta_P = 60\% = 0.6$

Top

Contents

1. *Thickness of pressure tank*

We know that thickness of pressure tank,

$$t = \frac{p \cdot d}{2\sigma_{t1} \cdot \eta} = \frac{3 \times 800}{2 \times 50 \times 1} = 24 \text{ mm} \quad \textbf{Ans.}$$

2. *Diameter and thickness of cylinder*

Let D = Diameter of cylinder, and

t_1 = Thickness of cylinder.

Since an allowance of 10 per cent of operating force F is provided for friction in the cylinder and packing, therefore total force to be produced by friction,

$$F_1 = F + \frac{10}{100} F = 1.1\, F = 1.1 \times 25 \times 10^3 = 27\,500 \text{ N}$$

Jacketed pressure vessel.

We know that there is a pressure drop of 0.2 N/mm^2 between the tank and cylinder, therefore pressure in the cylinder,

$$p_1 = \text{Pressure in tank} - \text{Pressure drop} = 3 - 0.2 = 2.8 \text{ N/mm}^2$$

and total force produced by friction (F_1),

$$27\,500 = \frac{\pi}{4} \times D^2 \times p_1 = 0.7854 \times D^2 \times 2.8 = 2.2\, D^2$$

$\therefore$ $D^2 = 27\,500 / 2.2 = 12\,500$ or $D = 112$ mm **Ans.**

We know that thickness of cylinder,

$$t_1 = \frac{p_1 \cdot D}{2\,\sigma_{tc}} = \frac{2.8 \times 112}{2 \times 30} = 5.2 \text{ mm} \quad \textbf{Ans.}$$

3. *Power output of the cylinder*

We know that stroke of the piston

= 450 mm = 0.45 m ...(Given)

and time required for working stroke

= 5 s ...(Given)

$\therefore$ Distance moved by the piston per second

$$= \frac{0.45}{5} = 0.09 \text{ m}$$

Top

We know that work done per second

$$= \text{Force} \times \text{Distance moved per second}$$

$$= 27\,500 \times 0.09 = 2475 \text{ N-m}$$

∴ Power output of the cylinder

$$= 2475 \text{ W} = 2.475 \text{ kW} \quad \textbf{Ans.} \qquad ...(\because 1 \text{ N-m/s} = 1 \text{ W})$$

4. Power of the motor

Since the working cycle repeats after every 30 seconds, therefore the power which is to be produced by the cylinder in 5 seconds is to be provided by the motor in 30 seconds.

∴ Power of the motor

$$= \frac{\text{Power of the cylinder}}{\eta_H \times \eta_P} \times \frac{5}{30} = \frac{2.475}{0.8 \times 0.6} \times \frac{5}{30} = 0.86 \text{ kW} \quad \textbf{Ans.}$$

7.6 Change in Dimensions of a Thin Cylindrical Shell due to an Internal Pressure

When a thin cylindrical shell is subjected to an internal pressure, there will be an increase in the diameter as well as the length of the shell.

Let l = Length of the cylindrical shell,

d = Diameter of the cylindrical shell,

t = Thickness of the cylindrical shell,

p = Intensity of internal pressure,

E = Young's modulus for the material of the cylindrical shell, and

μ = Poisson's ratio.

The increase in diameter of the shell due to an internal pressure is given by,

$$\delta d = \frac{p.d^2}{2\,t.E}\left(1 - \frac{\mu}{2}\right)$$

The increase in length of the shell due to an internal pressure is given by,

$$\delta l = \frac{p.d.l}{2\,t.E}\left(\frac{1}{2} - \mu\right)$$

It may be noted that the increase in diameter and length of the shell will also increase its volume. The increase in volume of the shell due to an internal pressure is given by

$$\delta V = \text{Final volume} - \text{Original volume} = \frac{\pi}{4}(d + \delta d)^2 (l + \delta l) - \frac{\pi}{4} \times d^2.l$$

$$= \frac{\pi}{4}(d^2.\delta l + 2\,d.l.\delta d) \qquad ...\text{(Neglecting small quantities)}$$

Example 7.4. *Find the thickness for a tube of internal diameter 100 mm subjected to an internal pressure which is 5/8 of the value of the maximum permissible circumferential stress. Also find the increase in internal diameter of such a tube when the internal pressure is 90 N/mm². Take E = 205 kN/mm² and μ = 0.29. Neglect longitudinal strain.*

Solution. Given : $p = 5/8 \times \sigma_{t1} = 0.625\,\sigma_{t1}$; $d = 100$ mm ; $p_1 = 90$ N/mm² ; $E = 205$ kN/mm² $= 205 \times 10^3$ N/mm² ; $\mu = 0.29$

Thickness of a tube

We know that thickness of a tube,

$$t = \frac{p\,.\,d}{2\,\sigma_{t1}} = \frac{0.625\,\sigma_{t1} \times 100}{2\,\sigma_{t1}} = 31.25 \text{ mm} \quad \textbf{Ans.}$$

Increase in diameter of a tube

We know that increase in diameter of a tube,

$$\delta d = \frac{p_1 \, d^2}{2 \, t.E}\left(1 - \frac{\mu}{2}\right) = \frac{90 \, (100)^2}{2 \times 31.25 \times 205 \times 10^3}\left[1 - \frac{0.29}{2}\right] \text{mm}$$

$$= 0.07 \, (1 - 0.145) = 0.06 \text{ mm}$$ **Ans.**

7.7 Thin Spherical Shells Subjected to an Internal Pressure

Consider a thin spherical shell subjected to an internal pressure as shown in Fig. 7.5.

Let V = Storage capacity of the shell,

p = Intensity of internal pressure,

d = Diameter of the shell,

t = Thickness of the shell,

σ_t = Permissible tensile stress for the shell material.

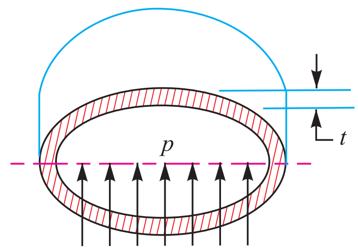


Fig. 7.5. Thin spherical shell.

In designing thin spherical shells, we have to determine 1. Diameter of the shell, and 2. Thickness of the shell.

1. *Diameter of the shell*

We know that the storage capacity of the shell,

$$V = \frac{4}{3} \times \pi \, r^3 = \frac{\pi}{6} \times d^3 \quad \text{or} \quad d = \left(\frac{6 \, V}{\pi}\right)^{1/3}$$

2. *Thickness of the shell*

As a result of the internal pressure, the shell is likely to rupture along the centre of the sphere. Therefore force tending to rupture the shell along the centre of the sphere or bursting force,

$$= \text{Pressure} \times \text{Area} = p \times \frac{\pi}{4} \times d^2 \qquad ...(i)$$

and resisting force of the shell

$$= \text{Stress} \times \text{Resisting area} = \sigma_t \times \pi \, d.t \qquad ...(ii)$$

Equating equations *(i)* and *(ii)*, we have

$$p \times \frac{\pi}{4} \times d^2 = \sigma_t \times \pi \, d.t$$

or

$$t = \frac{p.d}{4 \, \sigma_t}$$

If η is the efficiency of the circumferential joints of the spherical shell, then

$$t = \frac{p.d}{4 \, \sigma_t . \eta}$$

The Trans-Alaska Pipeline carries crude oil 1, 284 kilometres through Alaska. The pipeline is 1.2 metres in diameter and can transport 318 million litres of crude oil a day.

Example 7.5. *A spherical vessel 3 metre diameter is subjected to an internal pressure of 1.5 N/mm². Find the thickness of the vessel required if the maximum stress is not to exceed 90 MPa. Take efficiency of the joint as 75%.*

Solution. Given: d = 3 m = 3000 mm ; p = 1.5 N/mm² ; σ_t = 90 MPa = 90 N/mm² ; η = 75% = 0.75

Contents

We know that thickness of the vessel,

$$t = \frac{p.d}{4\,\sigma_t\,.\,\eta} = \frac{1.5 \times 3000}{4 \times 90 \times 0.75} = 16.7 \text{ say } 18 \text{ mm } \textbf{Ans.}$$

7.8 Change in Dimensions of a Thin Spherical Shell due to an Internal Pressure

Consider a thin spherical shell subjected to an internal pressure as shown in Fig. 7.5.

Let d = Diameter of the spherical shell,

t = Thickness of the spherical shell,

p = Intensity of internal pressure,

E = Young's modulus for the material of the spherical shell, and

μ = Poisson's ratio.

Increase in diameter of the spherical shell due to an internal pressure is given by,

$$\delta d = \frac{p.d^2}{4\,t.E}(1 - \mu) \qquad \ldots(i)$$

and increase in volume of the spherical shell due to an internal pressure is given by,

$$\delta V = \text{Final volume} - \text{Original volume} = \frac{\pi}{6}(d + \delta d)^3 - \frac{\pi}{6} \times d^3$$

$$= \frac{\pi}{6}(3d^2 \times \delta d) \qquad \ldots\text{(Neglecting higher terms)}$$

Substituting the value of δd from equation (*i*), we have

$$\delta V = \frac{3\pi d^2}{6}\left[\frac{p.d^2}{4\,t.E}(1 - \mu)\right] = \frac{\pi\, p\, d^4}{8\,t.E}(1 - \mu)$$

Example 7.6. *A seamless spherical shell, 900 mm in diameter and 10 mm thick is being filled with a fluid under pressure until its volume increases by 150 × 10^3 mm^3. Calculate the pressure exerted by the fluid on the shell, taking modulus of elasticity for the material of the shell as 200 kN/mm^2 and Poisson's ratio as 0.3.*

Solution. Given : d = 900 mm ; t = 10 mm ; $\delta V = 150 \times 10^3$ mm^3 ; E = 200 kN/mm^2 = 200 × 10^3 N/mm^2 ; μ = 0.3

Let p = Pressure exerted by the fluid on the shell.

We know that the increase in volume of the spherical shell (δV),

$$150 \times 10^3 = \frac{\pi\, p\, d^4}{8\, t\, E}(1 - \mu) = \frac{\pi\, p\,(900)^4}{8 \times 10 \times 200 \times 10^3}(1 - 0.3) = 90\,190\, p$$

$\therefore$ $p = 150 \times 10^3 / 90\,190 = 1.66$ N/mm^2 **Ans.**

7.9 Thick Cylindrical Shells Subjected to an Internal Pressure

When a cylindrical shell of a pressure vessel, hydraulic cylinder, gunbarrel and a pipe is subjected to a very high internal fluid pressure, then the walls of the cylinder must be made extremely heavy or thick.

In thin cylindrical shells, we have assumed that the tensile stresses are uniformly distributed over the section of the walls. But in the case of thick wall cylinders as shown in Fig. 7.6 (*a*), the stress over the section of the walls cannot be assumed to be uniformly distributed. They develop both tangential and radial stresses with values which are dependent upon the radius of the element under consideration. The distribution of stress in a thick cylindrical shell is shown in Fig. 7.6 (*b*) and (*c*). We see that the tangential stress is maximum at the inner surface and minimum at the outer surface of the shell. The radial stress is maximum at the inner surface and zero at the outer surface of the shell.

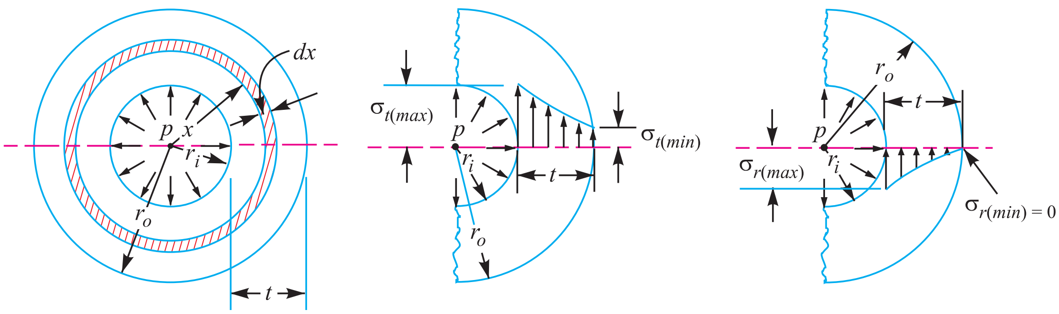
Top

Contents

In the design of thick cylindrical shells, the following equations are mostly used:

1. Lame's equation; **2.** Birnie's equation; **3.** Clavarino's equation; and **4.** Barlow's equation.

The use of these equations depends upon the type of material used and the end construction.



(*a*) Thick cylindrical shell. (*b*) Tangential stress distribution. (*c*) Radial stress distribution.

Fig. 7.6. Stress distribution in thick cylindrical shells subjected to internal pressure.

Let r_o = Outer radius of cylindrical shell,

r_i = Inner radius of cylindrical shell,

t = Thickness of cylindrical shell = $r_o - r_i$,

p = Intensity of internal pressure,

μ = Poisson's ratio,

σ_t = Tangential stress, and

σ_r = Radial stress.

All the above mentioned equations are now discussed, in detail, as below:

1. ***Lame's equation.*** Assuming that the longitudinal fibres of the cylindrical shell are equally strained, Lame has shown that the tangential stress at any radius x is,

$$\sigma_t = \frac{p_i (r_i)^2 - p_o (r_o)^2}{(r_o)^2 - (r_i)^2} + \frac{(r_i)^2 (r_o)^2}{x^2} \left[\frac{p_i - p_o}{(r_o)^2 - (r_i)^2} \right]$$

While designing a tanker, the pressure added by movement of the vehicle also should be considered.

Top

and radial stress at any radius x,

$$\sigma_r = \frac{p_i (r_i)^2 - p_o (r_o)^2}{(r_o)^2 - (r_i)^2} - \frac{(r_i)^2 (r_o)^2}{x^2}\left[\frac{p_i - p_o}{(r_o)^2 - (r_i)^2}\right]$$

Since we are concerned with the internal pressure ($p_i = p$) only, therefore substituting the value of external pressure, $p_o = 0$.

∴ Tangential stress at any radius x,

$$\sigma_t = \frac{p\,(r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 + \frac{(r_o)^2}{x^2}\right] \quad ...(i)$$

and radial stress at any radius x,

$$\sigma_r = \frac{p\,(r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 - \frac{(r_o)^2}{x^2}\right] \quad ...(ii)$$

We see that the tangential stress is always a tensile stress whereas the radial stress is a compressive stress. We know that the tangential stress is maximum at the inner surface of the shell (*i.e.* when $x = r_i$) and it is minimum at the outer surface of the shell (*i.e.* when $x = r_o$). Substituting the value of $x = r_i$ and $x = r_o$ in equation (*i*), we find that the *maximum tangential stress at the inner surface of the shell,

$$\sigma_{t(max)} = \frac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2}$$

and minimum tangential stress at the outer surface of the shell,

$$\sigma_{t(min)} = \frac{2\,p\,(r_i)^2}{(r_o)^2 - (r_i)^2}$$

We also know that the radial stress is maximum at the inner surface of the shell and zero at the outer surface of the shell. Substituting the value of $x = r_i$ and $x = r_o$ in equation (*ii*), we find that maximum radial stress at the inner surface of the shell,

$$\sigma_{r(max)} = -p \text{ (compressive)}$$

and minimum radial stress at the outer surface of the shell,

$$\sigma_{r(min)} = 0$$

In designing a thick cylindrical shell of brittle material (*e.g.* cast iron, hard steel and cast aluminium) with closed or open ends and in accordance with the maximum normal stress theory failure, the tangential stress induced in the cylinder wall,

$$\sigma_t = \sigma_{t(max)} = \frac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2}$$

Since $r_o = r_i + t$, therefore substituting this value of r_o in the above expression, we get

$$\sigma_t = \frac{p\,[(r_i + t)^2 + (r_i)^2]}{(r_i + t)^2 - (r_i)^2}$$

$$\sigma_t (r_i + t)^2 - \sigma_t (r_i)^2 = p\,(r_i + t)^2 + p\,(r_i)^2$$

$$(r_i + t)^2 (\sigma_t - p) = (r_i)^2 (\sigma_t + p)$$

$$\frac{(r_i + t)^2}{(r_i)^2} = \frac{\sigma_t + p}{\sigma_t - p}$$

* The maximum tangential stress is always greater than the internal pressure acting on the shell.

$$\frac{r_i + t}{r_i} = \sqrt{\frac{\sigma_t + p}{\sigma_t - p}} \quad \text{or} \quad 1 + \frac{t}{r_i} = \sqrt{\frac{\sigma_t + p}{\sigma_t - p}}$$

$$\therefore \quad \frac{t}{r_i} = \sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1 \quad \text{or} \quad t = r_i \left[\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1 \right] \quad ...(iii)$$

The value of σ_t for brittle materials may be taken as 0.125 times the ultimate tensile strength (σ_u).

We have discussed above the design of a thick cylindrical shell of brittle materials. In case of cylinders made of ductile material, Lame's equation is modified according to maximum shear stress theory.

According to this theory, the maximum shear stress at any point in a strained body is equal to one-half the algebraic difference of the maximum and minimum principal stresses at that point. We know that for a thick cylindrical shell,

Maximum principal stress at the inner surface,

$$\sigma_{t\,(max)} = \frac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2}$$

and minimum principal stress at the outer surface,

$$\sigma_{t(min)} = -p$$

$\therefore$ Maximum shear stress,

$$\tau = \tau_{max} = \frac{\sigma_{t(max)} - \sigma_{t(min)}}{2} = \frac{\dfrac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2} - (-p)}{2}$$

$$= \frac{p\,[(r_o)^2 + (r_i)^2] + p\,[(r_o)^2 - (r_i)^2]}{2[(r_o)^2 - (r_i)^2]} = \frac{2\,p\,(r_o)^2}{2[(r_o)^2 - (r_i)^2]}$$

$$= \frac{p\,(r_i + t)^2}{(r_i + t)^2 - (r_i)^2} \quad ...(\because r_o = r_i + t)$$

or

$$\tau(r_i + t)^2 - \tau(r_i)^2 = p(r_i + t)^2$$

$$(r_i + t)^2\,(\tau - p) = \tau(r_i)^2$$

$$\frac{(r_i + t)^2}{(r_i)^2} = \frac{\tau}{\tau - p}$$

$$\frac{r_i + t}{r_i} = \sqrt{\frac{\tau}{\tau - p}} \quad \text{or} \quad 1 + \frac{t}{r_i} = \sqrt{\frac{\tau}{\tau - p}}$$

$$\therefore \quad \frac{t}{r_i} = \sqrt{\frac{\tau}{\tau - p}} - 1 \quad \text{or} \quad t = r_i \left[\sqrt{\frac{\tau}{\tau - p}} - 1 \right] \quad ...(iv)$$

The value of shear stress (τ) is usually taken as one-half the tensile stress (σ_t). Therefore the above expression may be written as

$$t = r_i \left[\sqrt{\frac{\sigma_t}{\sigma_t - 2\,p}} - 1 \right] \quad ...(v)$$

From the above expression, we see that if the internal pressure (p) is equal to or greater than the allowable working stress (σ_t or τ), then no thickness of the cylinder wall will prevent failure. Thus, it is impossible to design a cylinder to withstand fluid pressure greater than the allowable working stress for a given material. This difficulty is overcome by using compound cylinders (See Art. 7.10).

2. ***Birnie's equation.*** In case of open-end cylinders (such as pump cylinders, rams, gun barrels etc.) made of ductile material (*i.e.* low carbon steel, brass, bronze, and aluminium alloys), the allowable stresses cannot be determined by means of maximum-stress theory of failure. In such cases, the maximum-strain theory is used. According to this theory, the failure occurs when the strain reaches a limiting value and Birnie's equation for the wall thickness of a cylinder is

$$t = r_i \left[\sqrt{\frac{\sigma_t + (1 - \mu)\ p}{\sigma_t - (1 + \mu)\ p}} - 1 \right]$$

The value of σ_t may be taken as 0.8 times the yield point stress (σ_y).

3. ***Clavarino's equation.*** This equation is also based on the maximum-strain theory of failure, but it is applied to closed-end cylinders (or cylinders fitted with heads) made of ductile material. According to this equation, the thickness of a cylinder,

Oil is frequently transported by ships called tankers. The larger tankers, such as this Acrco Alaska oil transporter, are known as supertankers. They can be hundreds of metres long.

$$t = r_i \left[\sqrt{\frac{\sigma_t + (1 - 2\mu)\ p}{\sigma_t - (1 + \mu)\ p}} - 1 \right]$$

In this case also, the value of σ_t may be taken as 0.8 σ_y.

4. ***Barlow's equation.*** This equation is generally used for high pressure oil and gas pipes. According to this equation, the thickness of a cylinder,

$$t = p.r_o / \sigma_t$$

For ductile materials, $\sigma_t = 0.8\ \sigma_y$ and for brittle materials $\sigma_t = 0.125\ \sigma_u$, where σ_u is the ultimate stress.

Example 7.7. *A cast iron cylinder of internal diameter 200 mm and thickness 50 mm is subjected to a pressure of 5 N/mm². Calculate the tangential and radial stresses at the inner, middle (radius = 125 mm) and outer surfaces.*

Solution. Given : d_i = 200 mm or r_i = 100 mm ; t = 50 mm ; p = 5 N/mm²

We know that outer radius of the cylinder,

$$r_o = r_i + t = 100 + 50 = 150 \text{ mm}$$

Contents

Tangential stresses at the inner, middle and outer surfaces

We know that the tangential stress at any radius x,

$$\sigma_t = \frac{p\,(r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 + \frac{(r_o)^2}{x^2}\right]$$

$\therefore$ Tangential stress at the inner surface (*i.e.* when $x = r_i = 100$ mm),

$$\sigma_{t(inner)} = \frac{p\,[(r_o)^2 + (r_i)^2]}{(r_o)^2 - (r_i)^2} = \frac{5\,[(150)^2 + (100)^2]}{(150)^2 - (100)^2} = 13 \text{ N/mm}^2 = 13 \text{ MPa } \textbf{Ans.}$$

Tangential stress at the middle surface (*i.e.* when $x = 125$ mm),

$$\sigma_{t(middle)} = \frac{5\,(100)^2}{(150)^2 - (100)^2}\left[1 + \frac{(150)^2}{(125)^2}\right] = 9.76 \text{ N/mm}^2 = 9.76 \text{ MPa } \textbf{Ans.}$$

and tangential stress at the outer surface (*i.e.* when $x = r_o = 150$ mm),

$$\sigma_{t(outer)} = \frac{2\,p\,(r_i)^2}{(r_o)^2 - (r_i)^2} = \frac{2 \times 5\,(100)^2}{(150)^2 - (100)^2} = 8 \text{ N/mm}^2 = 8 \text{ MPa } \textbf{Ans.}$$

Radial stresses at the inner, middle and outer surfaces

We know that the radial stress at any radius x,

$$\sigma_r = \frac{p\,(r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 - \frac{(r_o)^2}{x^2}\right]$$

$\therefore$ Radial stress at the inner surface (*i.e.* when $x = r_i = 100$ mm),

$$\sigma_{r(inner)} = -p = -5 \text{ N/mm}^2 = 5 \text{ MPa (compressive) } \textbf{Ans.}$$

Radial stress at the middle surface (*i.e.* when $x = 125$ mm)

$$\sigma_{r(middle)} = \frac{5\,(100)^2}{(150)^2 - (100)^2}\left[1 - \frac{(150)^2}{(125)^2}\right] = -1.76 \text{ N/mm}^2 = -1.76 \text{ MPa}$$

$$= 1.76 \text{ MPa (compressive) } \textbf{Ans.}$$

and radial stress at the outer surface (*i.e.* when $x = r_o = 150$ mm),

$$\sigma_{r(outer)} = 0 \textbf{ Ans.}$$

Example 7.8. *A hydraulic press has a maximum capacity of 1000 kN. The piston diameter is 250 mm. Calculate the wall thickness if the cylinder is made of material for which the permissible strength may be taken as 80 MPa. This material may be assumed as a brittle material.*

Hydraulic Press

Solution. Given : $W = 1000$ kN $= 1000 \times 10^3$ N ; $d = 250$ mm ; $\sigma_t = 80$ MPa $= 80$ N/mm²

First of all, let us find the pressure inside the cylinder (p). We know that load on the hydraulic press (W),

$$1000 \times 10^3 = \frac{\pi}{4} \times d^2 \times p = \frac{\pi}{4}\,(250)^2\,p = 49.1 \times 10^3 p$$

$\therefore$ $p = 1000 \times 10^3 / 49.1 \times 10^3 = 20.37$ N/mm²

Let r_i = Inside radius of the cylinder $= d/2 = 125$ mm

Top

We know that wall thickness of the cylinder,

$$t = r_i \left[\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1 \right] = 125 \left[\sqrt{\frac{80 + 20.37}{80 - 20.37}} - 1 \right] \text{mm}$$

$$= 125\,(1.297 - 1) = 37 \text{ mm } \textbf{Ans.}$$

Example 7.9. *A closed-ended cast iron cylinder of 200 mm inside diameter is to carry an internal pressure of 10 N/mm² with a permissible stress of 18 MPa. Determine the wall thickness by means of Lame's and the maximum shear stress equations. What result would you use? Give reason for your conclusion.*

Solution. Given : d_i = 200 mm or r_i = 100 mm ; p = 10 N/mm^2 ; σ_t = 18 MPa = 18 N/mm^2

According to Lame's equation, wall thickness of a cylinder,

$$t = r_i \left[\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1 \right] = 100 \left[\sqrt{\frac{80 + 10}{80 - 10}} - 1 \right] = 87 \text{ mm}$$

According to maximum shear stress equation, wall thickness of a cylinder,

$$t = r_i \left[\sqrt{\frac{\tau}{\tau - p}} - 1 \right]$$

We have discussed in Art. 7.9 [equation (*iv*)], that the shear stress (τ) is usually taken one-half the tensile stress (σ_t). In the present case, $\tau = \sigma_t / 2 = 18/2 = 9$ N/mm^2. Since τ is less than the internal pressure (p = 10 N/mm^2), therefore the expression under the square root will be negative. Thus no thickness can prevent failure of the cylinder. Hence it is impossible to design a cylinder to withstand fluid pressure greater than the allowable working stress for the given material. This difficulty is overcome by using compound cylinders as discussed in Art. 7.10.

Thus, we shall use a cylinder of wall thickness, t = 87 mm **Ans.**

Example 7.10. *The cylinder of a portable hydraulic riveter is 220 mm in diameter. The pressure of the fluid is 14 N/mm² by gauge. Determine suitable thickness of the cylinder wall assuming that the maximum permissible tensile stress is not to exceed 105 MPa.*

Solution. Given : d_i = 220 mm or r_i = 110 mm ; p = 14 N/mm^2 ; σ_t = 105 MPa = 105 N/mm^2

Since the pressure of the fluid is high, therefore thick cylinder equation is used.

Assuming the material of the cylinder as steel, the thickness of the cylinder wall (t) may be obtained by using Birnie's equation. We know that

$$t = r_i \left[\sqrt{\frac{\sigma_t + (1 - \mu)\,p}{\sigma_t - (1 + \mu)\,p}} - 1 \right]$$

$$= 110 \left[\sqrt{\frac{105 + (1 - 0.3)\,14}{105 - (1 + 0.3)\,14}} - 1 \right] = 16.5 \text{ mm } \textbf{Ans.}$$

...(Taking Poisson's ratio for steel, μ = 0.3)

Example 7.11. *The hydraulic cylinder 400 mm bore operates at a maximum pressure of 5 N/mm². The piston rod is connected to the load and the cylinder to the frame through hinged joints. Design: 1. cylinder, 2. piston rod, 3. hinge pin, and 4. flat end cover.*

The allowable tensile stress for cast steel cylinder and end cover is 80 MPa and for piston rod is 60 MPa.

Draw the hydraulic cylinder with piston, piston rod, end cover and O-ring.

Solution. Given : d_i = 400 mm or r_i = 200 mm ; p = 5 N/mm^2 ; σ_t = 80 MPa = 80 N/mm^2 ; σ_{tp} = 60 MPa = 60 N/mm^2

1. *Design of cylinder*

Let d_o = Outer diameter of the cylinder.

We know that thickness of cylinder,

$$t = r_i\left[\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1\right] = 200\left[\sqrt{\frac{80+5}{80-5}} - 1\right] \text{ mm}$$

$$= 200\,(1.06 - 1) = 12 \text{ mm } \textbf{Ans.}$$

∴ Outer diameter of the cylinder,

$$d_o = d_i + 2t = 400 + 2 \times 12 = 424 \text{ mm } \textbf{Ans.}$$

2. *Design of piston rod*

Let d_p = Diameter of the piston rod.

We know that the force acting on the piston rod,

$$F = \frac{\pi}{4}(d_i)^2\, p = \frac{\pi}{4}(400)^2\, 5 = 628\,400 \text{ N} \qquad ...(i)$$

We also know that the force acting on the piston rod,

$$F = \frac{\pi}{4}(d_i)^2\, \sigma_{tp} = \frac{\pi}{4}(d_p)^2\, 60 = 47.13\,(d_p)^2 \text{ N} \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$(d_p)^2 = 628\,400/47.13 = 13\,333.33 \quad \text{or} \quad d_p = 115.5 \text{ say } 116 \text{ mm } \textbf{Ans.}$$

3. *Design of the hinge pin*

Let d_h = Diameter of the hinge pin of the piston rod.

Since the load on the pin is equal to the force acting on the piston rod, and the hinge pin is in double shear, therefore

$$F = 2 \times \frac{\pi}{4}(d_h)^2\, \tau$$

$$628\,400 = 2 \times \frac{\pi}{4}(d_h)^2\, 45 = 70.7\,(d_h)^2 \qquad ...(\text{Taking } \tau = 45 \text{ N/mm}^2)$$

$$\therefore \quad (d_h)^2 = 628\,400 / 70.7 = 8888.3 \quad \text{or} \quad d_h = 94.3 \text{ say } 95 \text{ mm } \textbf{Ans.}$$

When the cover is hinged to the cylinder, we can use two hinge pins only diametrically opposite to each other. Thus the diameter of the hinge pins for cover,

$$d_{hc} = \frac{d_h}{2} = \frac{95}{2} = 47.5 \text{ mm } \textbf{Ans.}$$

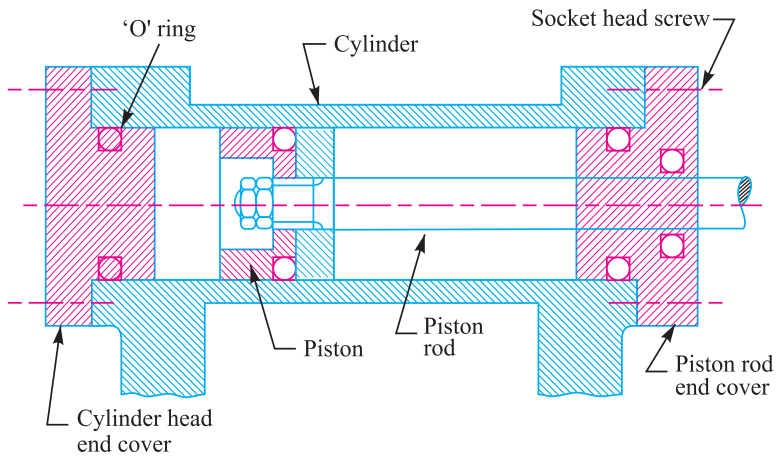


Fig. 7.7

4. *Design of the flat end cover*

Let t_c = Thickness of the end cover.

We know that force on the end cover,

$$F = d_i \times t_c \times \sigma_t$$

$$628\ 400 = 400 \times t_c \times 80 = 32 \times 10^3\ t_c$$

$$\therefore \quad t_c = 628\ 400 / 32 \times 10^3 = 19.64 \text{ say } 20 \text{ mm}$$ **Ans.**

The hydraulic cylinder with piston, piston rod, end cover and *O*-ring is shown in Fig. 7.7.

7.10 Compound Cylindrical Shells

According to Lame's equation, the thickness of a cylindrical shell is given by

$$t = r_i \left(\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1 \right)$$

From this equation, we see that if the internal pressure (p) acting on the shell is equal to or greater than the allowable working stress (σ_t) for the material of the shell, then no thickness of the shell will prevent failure. Thus it is impossible to design a cylinder to withstand internal pressure equal to or greater than the allowable working stress.

This difficulty is overcome by inducing an initial compressive stress on the wall of the cylindrical shell. This may be done by the following two methods:

1. By using compound cylindrical shells, and
2. By using the theory of plasticity.

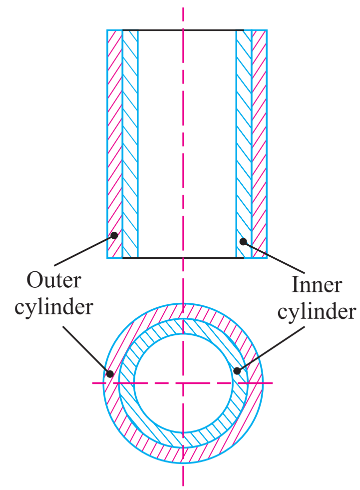


Fig. 7.8. Compound cylindrical shell.

In a compound cylindrical shell, as shown in Fig. 7.8, the outer cylinder (having inside diameter smaller than the outside diameter of the inner cylinder) is shrunk fit over the inner cylinder by heating and cooling. On cooling, the contact pressure is developed at the junction of the two cylinders, which induces compressive tangential stress in the material of the inner cylinder and tensile tangential stress in the material of the outer cylinder. When the cylinder is loaded, the compressive stresses are first relieved and then tensile stresses are induced. Thus, a compound cylinder is effective in resisting higher internal pressure than a single cylinder with the same overall dimensions. The principle of compound cylinder is used in the design of gun tubes.

In the theory of plasticity, a temporary high internal pressure is applied till the plastic stage is reached near the inside of the cylinder wall. This results in a residual compressive stress upon the removal of the internal pressure, thereby making the cylinder more effective to withstand a higher internal pressure.

7.11 Stresses in Compound Cylindrical Shells

Fig. 7.9 (*a*) shows a compound cylindrical shell assembled with a shrink fit. We have discussed in the previous article that when the outer cylinder is shrunk fit over the inner cylinder, a contact pressure (p) is developed at junction of the two cylinders (*i.e.* at radius r_2) as shown in Fig. 7.9 (*b*) and (*c*). The stresses resulting from this pressure may be easily determined by using Lame's equation.

According to this equation (See Art. 7.9), the tangential stress at any radius x is

$$\sigma_t = \frac{p_i (r_i)^2 - p_o (r_o)^2}{(r_o)^2 - (r_i)^2} + \frac{(r_i)^2 (r_o)^2}{x^2} \left[\frac{p_i - p_o}{(r_o)^2 - (r_i)^2} \right] \quad \text{...}(i)$$

Contents

and radial stress at any radius x,

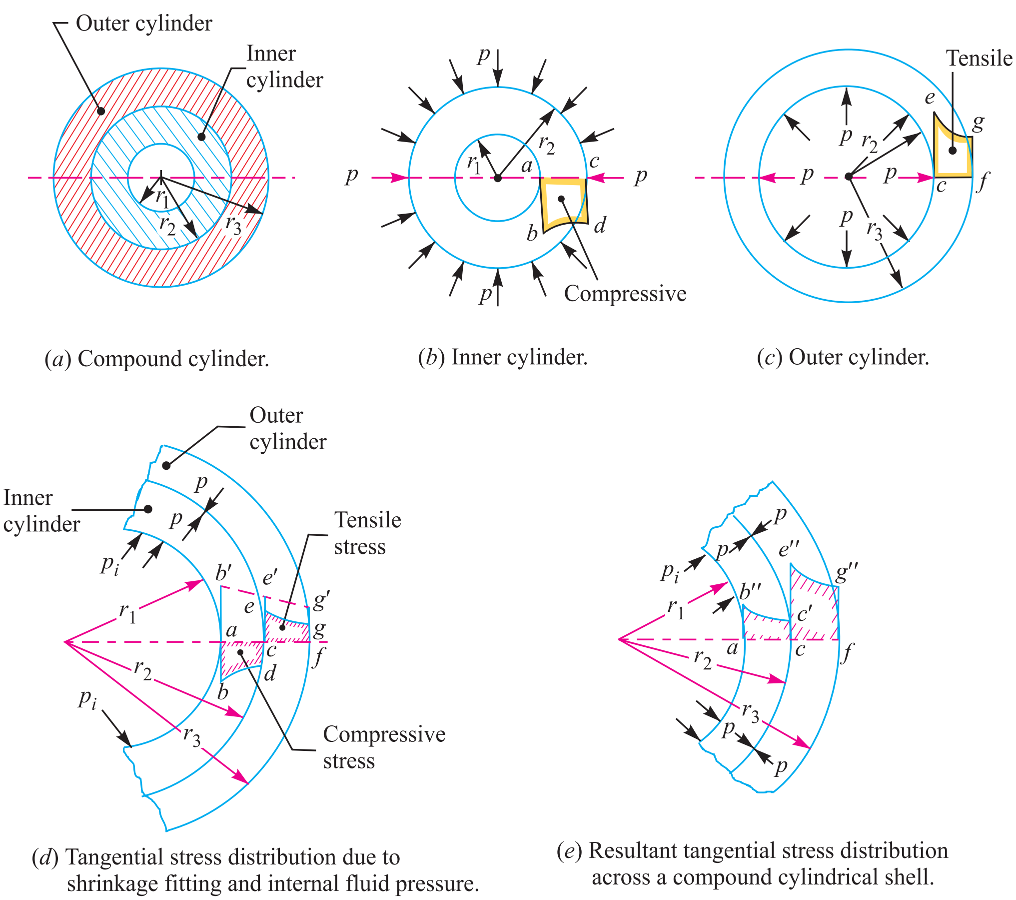
$$\sigma_r = \frac{p_i (r_i)^2 - p_o (r_o)^2}{(r_o)^2 - (r_i)^2} - \frac{(r_i)^2 (r_o)^2}{x^2}\left[\frac{p_i - p_o}{(r_o)^2 - (r_i)^2}\right] \quad \text{...(ii)}$$

Considering the external pressure only,

$$\sigma_t = \frac{-p_o (r_o)^2}{(r_o)^2 - (r_i)^2}\left[1 + \frac{(r_i)^2}{x^2}\right] \quad \text{...(iii)}$$

...[Substituting $p_i = 0$ in equation (*i*)]

and
$$\sigma_r = \frac{-p_o (r_o)^2}{(r_o)^2 - (r_i)^2}\left[1 - \frac{(r_i)^2}{x^2}\right] \quad \text{...(iv)}$$



(*a*) Compound cylinder. (*b*) Inner cylinder. (*c*) Outer cylinder.

(*d*) Tangential stress distribution due to shrinkage fitting and internal fluid pressure.

(*e*) Resultant tangential stress distribution across a compound cylindrical shell.

Fig. 7.9. Stresses in compound cylindrical shells.

Considering the internal pressure only,

$$\sigma_t = \frac{p_i (r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 + \frac{(r_o)^2}{x^2}\right] \quad \text{...(v)}$$

...[Substituting $p_o = 0$ in equation (*i*)]

Top

and $$\sigma_r = \frac{p_i\,(r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 - \frac{(r_o)^2}{x^2}\right] \qquad \text{...}(vi)$$

Since the inner cylinder is subjected to an external pressure (p) caused by the shrink fit and the outer cylinder is subjected to internal pressure (p), therefore from equation (*iii*), we find that the tangential stress at the inner surface of the inner cylinder,

$$\sigma_{t1} = \frac{-\,p\,(r_2)^2}{(r_2)^2 - (r_1)^2}\left[1 + \frac{(r_1)^2}{(r_1)^2}\right] = \frac{-\,2\,p\,(r_2)^2}{(r_2)^2 - (r_1)^2} \text{ (compressive)} \qquad \text{...}(vii)$$

... [Substituting $p_o = p$, $x = r_1$, $r_o = r_2$ and $r_i = r_1$]

This stress is compressive and is shown by *ab* in Fig. 7.9 (*b*).

Radial stress at the inner surface of the inner cylinder,

$$\sigma_{r1} = \frac{-\,p\,(r_2)^2}{(r_2)^2 - (r_1)^2}\left[1 - \frac{(r_1)^2}{(r_1)^2}\right] = 0 \qquad \text{...[From equation } (iv)]$$

Similarly from equation (*iii*), we find that tangential stress at the outer surface of the inner cylinder,

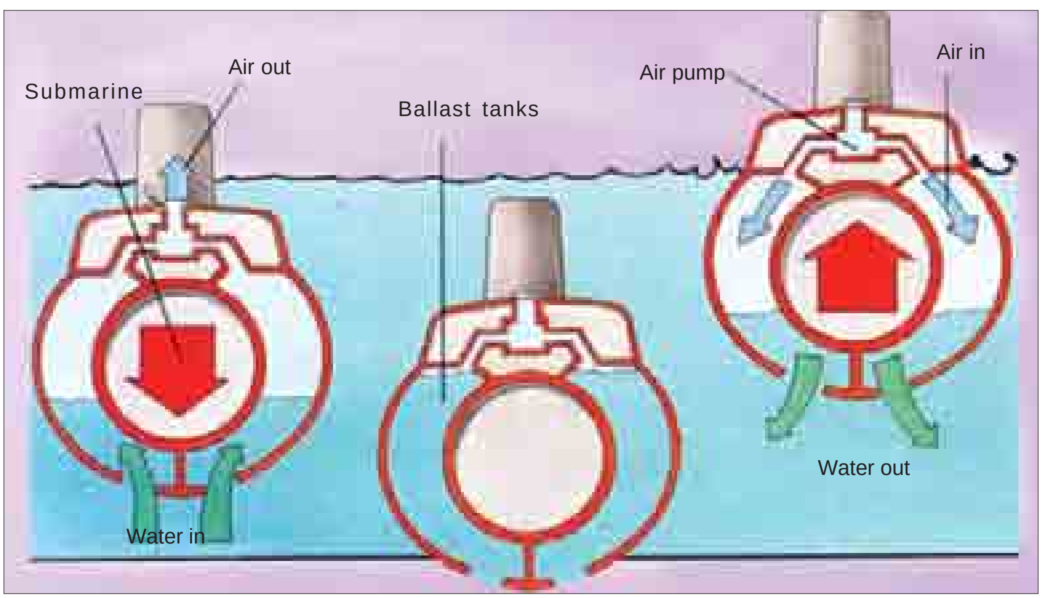
$$\sigma_{t2} = \frac{-\,p\,(r_2)^2}{(r_2)^2 - (r_1)^2}\left[1 + \frac{(r_1)^2}{(r_2)^2}\right] = \frac{-\,p\,[(r_2)^2 + (r_1)^2]}{(r_2)^2 - (r_1)^2} \text{ (compressive)} \qquad \text{...}(viii)$$

...[Substituting $p_o = p$, $x = r_2$, $r_o = r_2$ and $r_i = r_1$]

This stress is compressive and is shown by *cd* in Fig. 7.9 (*b*).

Radial stress at the outer surface of the inner cylinder,

$$\sigma_{r2} = \frac{-\,p\,(r_2)^2}{(r_2)^2 - (r_1)^2}\left[1 - \frac{(r_1)^2}{(r_2)^2}\right] = -\,p$$



Submarines consist of an airtight compartment surrounded by ballast tanks. The submarine dives by filling these tanks with water or air. Its neutral buoyancy ensures that it neither floats nor sinks.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Contents

Now let us consider the outer cylinder subjected to internal pressure (p). From equation **(*v*)**, we find that the tangential stress at the inner surface of the outer cylinder,

$$\sigma_{t3} = \frac{p\,(r_2)^2}{(r_3)^2 - (r_2)^2}\left[1 + \frac{(r_3)^2}{(r_2)^2}\right] = \frac{p\,[(r_3)^2 + (r_2)^2]}{(r_3)^2 - (r_2)^2} \text{ (tensile)} \quad ...(ix)$$

...[Substituting $p_i = p$, $x = r_2$, $r_o = r_3$ and $r_i = r_2$]

This stress is tensile and is shown by *ce* in Fig. 7.9 (*c*).

Radial stress at the inner surface of the outer cylinder,

$$\sigma_{r3} = \frac{p\,(r_2)^2}{(r_3)^2 - (r_2)^2}\left[1 - \frac{(r_3)^2}{(r_2)^2}\right] = -\,p \quad \text{...[From equation } (vi)]$$

Similarly from equation (*v*), we find that the tangential stress at the outer surface of the outer cylinder,

$$\sigma_{t4} = \frac{p\,(r_2)^2}{(r_3)^2 - (r_2)^2}\left[1 + \frac{(r_3)^2}{(r_3)^2}\right] = \frac{2\,p\,(r_2)^2}{(r_3)^2 - (r_2)^2} \text{ (tensile)} \quad ...(x)$$

...[Substituting $p_i = p$, $x = r_3$, $r_o = r_3$ and $r_i = r_2$]

This stress is tensile and is shown by *fg* in Fig. 7.9 (*c*).

Radial stress at the outer surface of the outer cylinder,

$$\sigma_{r4} = \frac{p\,(r_2)^2}{(r_3)^2 - (r_2)^2}\left[1 - \frac{(r_3)^2}{(r_3)^2}\right] = 0$$

The equations **(*vii*)** to **(*x*)** cannot be solved until the contact pressure (p) is known. In obtaining a shrink fit, the outside diameter of the inner cylinder is made larger than the inside diameter of the outer cylinder. This difference in diameters is called the **interference** and is the deformation which the two cylinders must experience. Since the diameters of the cylinders are usually known, therefore the deformation should be calculated to find the contact pressure.

Submarine is akin a to pressure vessel. CAD and CAM were used to design and manufacture this French submarine.

Let δ_o = Increase in inner radius of the outer cylinder,

δ_i = Decrease in outer radius of the inner cylinder,

E_o = Young's modulus for the material of the outer cylinder,

E_i = Young's modulus for the material of the inner cylinder, and

μ = Poisson's ratio.

We know that the tangential strain in the outer cylinder at the inner radius (r_2),

$$\varepsilon_{to} = \frac{\text{Change in circumference}}{\text{Original circumference}} = \frac{2\pi\,(r_2 + \delta_o) - 2\pi\,r_2}{2\pi\,r_2} = \frac{\delta_o}{r_2} \quad ...(xi)$$

Also the tangential strain in the outer cylinder at the inner radius (r_2),

$$\varepsilon_{to} = \frac{\sigma_{to}}{E_o} - \frac{\mu.\sigma_{ro}}{E_o} \quad ...(xii)$$

Top

Contents

We have discussed above that the tangential stress at the inner surface of the outer cylinder (or at the contact surfaces),

$$\sigma_{to} = \sigma_{t3} = \frac{p\,[(r_3)^2 + (r_2)^2]}{(r_3)^2 - (r_2)^2} \qquad \text{...[From equation } (ix)]$$

and radial stress at the inner surface of the outer cylinder (or at the contact surfaces),

$$\sigma_{ro} = \sigma_{r3} = -p$$

Substituting the value of σ_{to} and σ_{ro} in equation (*xii*), we get

$$\varepsilon_{to} = \frac{p\,[(r_3)^2 + (r_2)^2]}{E_o\,[(r_3)^2 - (r_2)^2]} + \frac{\mu.p}{E_o} = \frac{p}{E_o}\left[\frac{(r_3)^2 + (r_2)^2}{(r_3)^2 - (r_2)^2} + \mu\right] \qquad \text{...}(xiii)$$

From equations (*xi*) and (*xiii*),

$$\delta_o = \frac{p.r_2}{E_o}\left[\frac{(r_3)^2 + (r_2)^2}{(r_3)^2 - (r_2)^2} + \mu\right] \qquad \text{...}(xiv)$$

Similarly, we may find that the decrease in the outer radius of the inner cylinder,

$$\delta_i = \frac{-\,p.r_2}{E_i}\left[\frac{(r_2)^2 + (r_1)^2}{(r_2)^2 - (r_1)^2} - \mu\right] \qquad \text{...}(xv)$$

∴ Difference in radius,

$$\delta_r = \delta_o - \delta_i = \frac{p.r_2}{E_o}\left[\frac{(r_3)^2 + (r_2)^2}{(r_3)^2 - (r_2)^2} + \mu\right] + \frac{p.r_2}{E_i}\left[\frac{(r_2)^2 + (r_1)^2}{(r_2)^2 - (r_1)^2} - \mu\right]$$

If both the cylinders are of the same material, then $E_o = E_i = E$. Thus the above expression may be written as

$$\delta_r = \frac{p.r_2}{E}\left[\frac{(r_3)^2 + (r_2)^2}{(r_3)^2 - (r_2)^2} + \frac{(r_2)^2 + (r_1)^2}{(r_2)^2 - (r_1)^2}\right]$$

$$= \frac{p.r_2}{E}\left[\frac{[(r_3)^2 + (r_2)^2]\,[(r_2)^2 - (r_1)^2] + [(r_2)^2 + (r_1)^2]\,[(r_3)^2 - (r_2)^2]}{[(r_3)^2 - (r_2)^2]\,[(r_2)^2 - (r_1)^2]}\right]$$

$$= \frac{p.r_2}{E}\left[\frac{2(r_2)^2\,[(r_3)^2 - (r_1)^2]}{[(r_3)^2 - (r_2)^2]\,[(r_2)^2 - (r_1)^2]}\right]$$

or

$$p = \frac{E.\delta_r}{r_2}\left[\frac{[(r_3)^2 - (r_2)^2]\,[(r_2)^2 - (r_1)^2]}{2(r_2)^2\,[(r_3)^2 - (r_1)^2]}\right]$$

Substituting this value of p in equations (*vii*) to (*x*), we may obtain the tangential stresses at the various surfaces of the compound cylinder.

Now let us consider the compound cylinder subjected to an internal fluid pressure (p_i). We have discussed above that when the compound cylinder is subjected to internal pressure (p_i), then the tangential stress at any radius (x) is given by

$$\sigma_t = \frac{p_i\,(r_i)^2}{(r_o)^2 - (r_i)^2}\left[1 + \frac{(r_o)^2}{x^2}\right]$$

∴ Tangential stress at the inner surface of the inner cylinder,

$$\sigma_{t5} = \frac{p_i\,(r_1)^2}{(r_3)^2 - (r_1)^2}\left[1 + \frac{(r_3)^2}{(r_1)^2}\right] = \frac{p_i\,[(r_3)^2 + (r_1)^2]}{(r_3)^2 - (r_1)^2} \text{ (tensile)}$$

... [Substituting $x = r_1$, $r_o = r_3$ and $r_i = r_1$]

Top

This stress is tensile and is shown by *ab'* in Fig. 7.9 (*d*).

Tangential stress at the outer surface of the inner cylinder or inner surface of the outer cylinder,

$$\sigma_{t6} = \frac{p_i (r_1)^2}{(r_3)^2 - (r_1)^2}\left[1 + \frac{(r_3)^2}{(r_2)^2}\right] = \frac{p_i (r_1)^2}{(r_2)^2}\left[\frac{(r_3)^2 + (r_2)^2}{(r_3)^2 - (r_1)^2}\right] \text{(tensile)}$$

... [Substituting $x = r_2$, $r_o = r_3$ and $r_i = r_1$]

This stress is tensile and is shown by *ce'* in Fig. 7.9 (*d*),

and tangential stress at the outer surface of the outer cylinder,

$$\sigma_{t7} = \frac{p_i (r_1)^2}{(r_3)^2 - (r_1)^2}\left[1 + \frac{(r_3)^2}{(r_3)^2}\right] = \frac{2\, p_i (r_1)^2}{(r_3)^2 - (r_1)^2} \text{ (tensile)}$$

...[Substituting $x = r_3$, $r_o = r_3$ and $r_i = r_1$]

This stress is tensile and is shown by *fg'* in Fig. 7.9 (*d*).

Now the resultant stress at the inner surface of the compound cylinder,

$$\sigma_{ti} = \sigma_{t1} + \sigma_{t5} \quad \text{or} \quad ab' - ab$$

This stress is tensile and is shown by *ab''* in Fig. 7.9 (*e*).

Resultant stress at the outer surface of the inner cylinder

$$= \sigma_{t2} + \sigma_{t6} \quad \text{or} \quad ce' - cd \text{ or } cc'$$

Resultant stress at the inner surface of the outer cylinder

$$= \sigma_{t3} + \sigma_{t6} \quad \text{or} \quad ce + ce' \text{ or } c'e''$$

∴ Total resultant stress at the mating or contact surface,

$$\sigma_{tm} = \sigma_{t2} + \sigma_{t6} + \sigma_{t3} + \sigma_{t6}$$

This stress is tensile and is shown by *ce''* in Fig. 7.9 (*e*),

and resultant stress at the outer surface of the outer cylinder,

$$\sigma_{to} = \sigma_{t4} + \sigma_{t7} \quad \text{or} \quad fg + fg'$$

This stress is tensile and is shown by *fg''* in Fig. 7.9 (*e*).

Example 7.12. *The hydraulic press, having a working pressure of water as 16 N/mm² and exerting a force of 80 kN is required to press materials upto a maximum size of 800 mm × 800 mm and 800 mm high, the stroke length is 80 mm. Design and draw the following parts of the press : 1. Design of ram; 2. Cylinder; 3. Pillars; and 4. Gland.*

Solution. Given: $p = 16$ N/mm^2 ; $F = 80$ kN $= 80 \times 10^3$ N

The hydraulic press is shown in Fig. 7.10.

1. *Design of ram*

Let d_r = Diameter of ram.

We know that the maximum force to be exerted by the ram (F),

$$80 \times 10^3 = \frac{\pi}{4}(d_r)^2 p = \frac{\pi}{4}(d_r)^2\, 16 = 12.57\,(d_r)^2$$

$$\therefore \quad (d_r)^2 = 80 \times 10^3/12.57 = 6364 \quad \text{or} \quad d_r = 79.8 \text{ say } 80 \text{ mm}$$ **Ans.**

In case the ram is made hollow in order to reduce its weight, then it can be designed as a thick cylinder subjected to external pressure. We have already discussed in Art. 7.11 that according to Lame's equation, maximum tangential stress (considering external pressure only) is

Contents

$$\sigma_{t(max)} = \frac{-p_o (d_{ro})^2}{(d_{ro})^2 - (d_{ri})^2}\left[1 + \frac{(d_{ri})^2}{(d_{ro})^2}\right] = -p_o\left[\frac{(d_{ro})^2 + (d_{ri})^2}{(d_{ro})^2 - (d_{ri})^2}\right] \text{(compressive)}$$

and maximum radial stress,

$$\sigma_{r(max)} = -p_o \text{ (compressive)}$$

where d_{ro} = Outer diameter of ram = d_r = 80 mm

d_{ri} = Inner diameter of ram, and

p_o = External pressure = p = 16 N/mm^2 ...(Given)

Now according to maximum shear stress theory for ductile materials, maximum shear stress is

$$\tau_{max} = \frac{\sigma_{t(max)} - \sigma_{r(max)}}{2} = \frac{-p_o\left[\frac{(d_{ro})^2 + (d_{ri})^2}{(d_{ro})^2 - (d_{ri})^2}\right] - (-p_o)}{2}$$

$$= -p_o\left[\frac{(d_{ri})^2}{(d_{ro})^2 - (d_{ri})^2}\right]$$

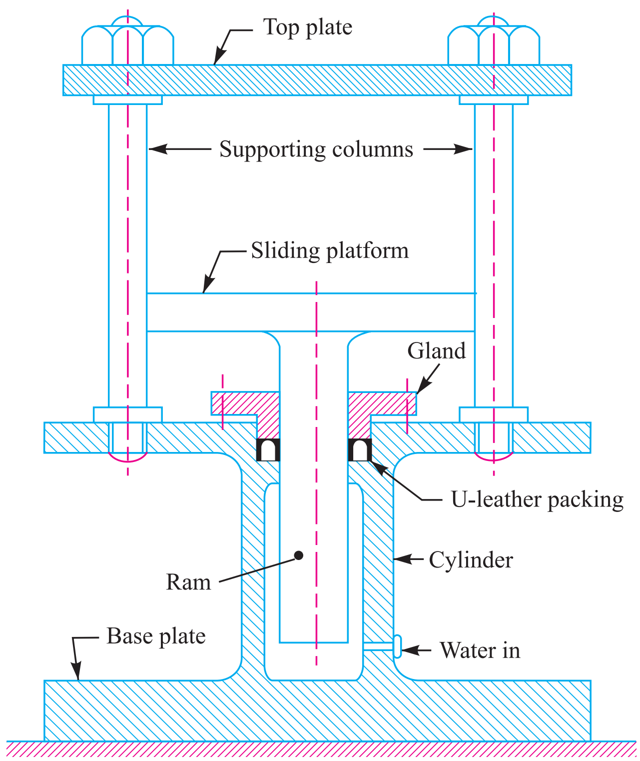


Fig. 7.10. Hydraulic press.

Since the maximum shear stress is one-half the maximum principal stress (which is compressive), therefore

$$\sigma_c = 2\,\tau_{max} = 2\,p_o\left[\frac{(d_{ri})^2}{(d_{ro})^2 - (d_{ri})^2}\right]$$

The ram is usually made of mild steel for which the compressive stress may be taken as 75 N/mm^2. Substituting this value of stress in the above expression, we get

Top

$$75 = 2 \times 16 \left[\frac{(d_{ri})^2}{(80)^2 - (d_{ri})^2} \right] = \frac{32\,(d_{ri})^2}{6400 - (d_{ri})^2}$$

or $$\frac{(d_{ri})^2}{6400 - (d_{ri})^2} = \frac{75}{32} = 2.34$$

$$(d_{ri})^2 = 2.34\,[6400 - (d_{ri})^2] = 14\,976 - 2.34\,(d_{ri})^2$$

$$3.34\,(d_{ri})^2 = 14\,976 \quad \text{or} \quad (d_{ri})^2 = 14\,976/3.34 = 4484$$

∴ d_{ri} = 67 mm **Ans.**

and $d_{ro} = d_r$ = 80 mm **Ans.**

2. Design of cylinder

Let d_{ci} = Inner diameter of cylinder, and

d_{co} = Outer diameter of cylinder.

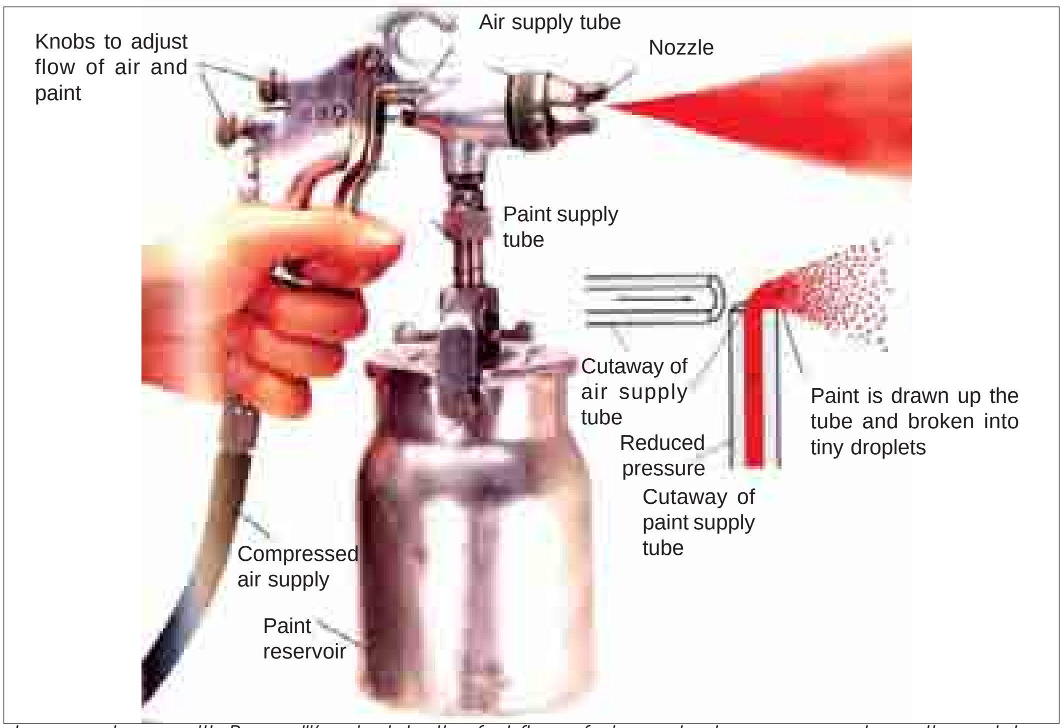
Assuming a clearance of 15 mm between the ram and the cylinder bore, therefore inner diameter of the cylinder,

$d_{ci} = d_{ro}$ + Clearance = 80 + 15 = 95 mm **Ans.**

The cylinder is usually made of cast iron for which the tensile stress may be taken as 30 N/mm^2. According to Lame's equation, we know that wall thickness of a cylinder,

$$t = \frac{d_{ci}}{2}\left[\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1\right] = \frac{95}{2}\left[\sqrt{\frac{30 + 16}{30 - 16}} - 1\right] \text{ mm}$$

$$= 47.5\,(1.81 - 1) = 38.5 \text{ say } 40 \text{ mm}$$



In accordance with Bernoulli's principle, the fast flow of air creates low pressure above the paint tube, sucking paint upwards into the air steam.

Note : This picture is given as additional information and is not a direct example of the current chapter.

and outside diameter of the cylinder,

$$d_{co} = d_{ci} + 2\,t = 95 + 2 \times 40 = 175 \text{ mm}$$ **Ans.**

3. *Design of pillars*

Let d_p = Diameter of the pillar.

The function of the pillars is to support the top plate and to guide the sliding plate. When the material is being pressed, the pillars will be under direct tension. Let there are four pillars and the load is equally shared by these pillars.

∴ Load on each pillar

$$= 80 \times 10^3/4 = 20 \times 10^3 \text{ N} \quad \text{...}(i)$$

We know that load on each pillar

$$= \frac{\pi}{4} (d_p)^2 \sigma_t = \frac{\pi}{4} (d_p)^2\, 75 = 58.9\, (d_p)^2 \quad \text{...}(ii)$$

From equations ***(i)*** and ***(ii)***,

$$(d_p)^2 = 20 \times 10^3/58.9 = 340 \quad \text{or} \quad d_p = 18.4 \text{ mm}$$

From fine series of metric threads, let us adopt the threads on pillars as M 20 × 1.5 having major diameter as 20 mm and core diameter as 18.16 mm. **Ans.**

4. *Design of gland*

The gland is shown in Fig 7.11. The width (w) of the U-leather packing for a ram is given empirically as $2\sqrt{d_r}$ to $2.5\sqrt{d_r}$, where d_r is the diameter (outer) of the ram in mm. Let us take width of the packing as $2.2\sqrt{d_r}$.

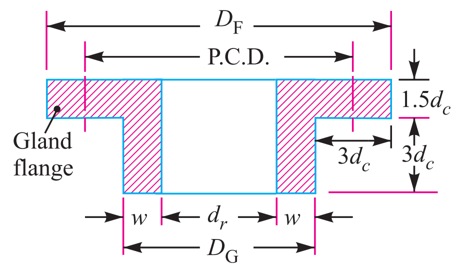


Fig. 7.11

∴ Width of packing,

$$w = 2.2\sqrt{80} = 19.7 \text{ say } 20 \text{ mm}$$ **Ans.**

and outer diameter of gland,

$$D_G = d_r + 2\,w = 80 + 2 \times 20 = 120 \text{ mm}$$ **Ans.**

We know that total upward load on the gland

= Area of gland exposed to fluid pressure × Fluid pressure

$$= \pi\,(d_r + w)\,w.p = \pi\,(80 + 20)\,20 \times 16 = 100\,544 \text{ N}$$

Let us assume that 8 studs equally spaced on the pitch circle of the gland flange are used for holding down the gland.

∴ Load on each stud = 100 544 / 8 = 12 568 N

If d_c is the core diameter of the stud and σ_t is the permissible tensile stress for the stud material, then

Load on each stud,

$$12\,568 = \frac{\pi}{4} (d_c)^2 \sigma_t = \frac{\pi}{4} (d_c)^2\, 75 = 58.9\, (d_c)^2 \quad \text{... (Taking } \sigma_t = 75 \text{ N/mm}^2)$$

$$\therefore \quad (d_c)^2 = 12\,568 / 58.9 = 213.4 \quad \text{or} \quad d_c = 14.6 \text{ mm}$$

From fine series of metric threads, let us adopt the studs of size M 18 × 1.5 having major diameter as 18 mm and core diameter (d_c) as 16.16 mm. **Ans.**

The other dimensions for the gland are taken as follows:

Contents

Pitch circle diameter of the gland flange,

$$\text{P.C.D.} = D_G + 3\, d_c = 120 + 3 \times 16.16 = 168.48 \text{ or } 168.5 \text{ mm } \textbf{Ans.}$$

Outer diameter of the gland flange,

$$D_F = D_G + 6\, d_c = 120 + 6 \times 16.16 = 216.96 \quad \text{or} \quad 217 \text{ mm } \textbf{Ans.}$$

and thickness of the gland flange = $1.5\, d_c = 1.5 \times 16.16 = 24.24$ or 24.5 mm **Ans.**

A long oil tank.

Example 7.13. *A steel tube 240 mm external diameter is shrunk on another steel tube of 80 mm internal diameter. After shrinking, the diameter at the junction is 160 mm. Before shrinking, the difference of diameters at the junction was 0.08 mm. If the Young's modulus for steel is 200 GPa, find: 1. tangential stress at the outer surface of the inner tube; 2. tangential stress at the inner surface of the outer tube ; and 3. radial stress at the junction.*

Solution. Given: $d_3 = 240$ mm or $r_3 = 120$ mm ; $d_1 = 80$ mm or $r_1 = 40$ mm ; $d_2 = 160$ mm or $r_2 = 80$ mm ; $\delta_d = 0.08$ mm or $\delta_r = 0.04$ mm ; $E = 200$ GPa $= 200$ kN/mm^2 $= 200 \times 10^3$ N/mm^2

First of all, let us find the pressure developed at the junction. We know that the pressure developed at the junction,

$$p = \frac{E.\delta_r}{r_2}\left[\frac{[(r_3)^2 - (r_2)^2]\,[(r_2)^2 - (r_1)^2]}{2\,(r_2)^2\,[(r_3)^2 - (r_1)^2]}\right]$$

$$= \frac{200 \times 10^3 \times 0.04}{80}\left[\frac{[(120)^2 - (80)^2]\,[(80)^2 - (40)^2]}{2 \times (80)^2\,[(120)^2 - (40)^2]}\right]$$

$$= 100 \times 0.234 = 23.4 \text{ N/mm}^2 \textbf{ Ans.}$$

1. ***Tangential stress at the outer surface of the inner tube***

We know that the tangential stress at the outer surface of the inner tube,

$$\sigma_{ti} = \frac{-\,p\,[(r_2)^2 + (r_1)^2]}{(r_2)^2 - (r_1)^2} = \frac{-\,23.4\,[(80)^2 + (40)^2]}{(80)^2 - (40)^2} = -\,39 \text{ N/mm}^2$$

$$= 39 \text{ MPa (compressive) } \textbf{Ans.}$$

Top

Contents

2. *Tangential stress at the inner surface of the outer tube*

We know that the tangential stress at the inner surface of the outer tube,

$$\sigma_{to} = \frac{-\,p\,[(r_3)^2 + (r_2)^2]}{(r_3)^2 - (r_2)^2} = \frac{23.4\,[(120)^2 + (80)^2]}{(120)^2 - (80)^2} = 60.84\ \text{N/mm}^2$$

$= 60.84$ MPa **Ans.**

3. *Radial stress at the junction*

We know that the radial stress at the junction, (*i.e.* at the inner radius of the outer tube),

$$\sigma_{ro} = -p = -23.4\ \text{N/mm}^2 = 23.4\ \text{MPa (compressive)}$$ **Ans.**

Example 7.14. *A shrink fit assembly, formed by shrinking one tube over another, is subjected to an internal pressure of 60 N/mm^2. Before the fluid is admitted, the internal and the external diameters of the assembly are 120 mm and 200 mm and the diameter at the junction is 160 mm. If after shrinking on, the contact pressure at the junction is 8 N/mm^2, determine using Lame's equations, the stresses at the inner, mating and outer surfaces of the assembly after the fluid has been admitted.*

Solution. Given : $p_i = 60$ N/mm^2 ; $d_1 = 120$ mm or $r_1 = 60$ mm ; $d_3 = 200$ mm or $r_3 = 100$ mm ; $d_2 = 160$ mm or $r_2 = 80$ mm ; $p = 8$ N/mm^2

First of all, let us find out the stresses induced in the assembly due to contact pressure at the junction (p).

We know that the tangential stress at the inner surface of the inner tube,

$$\sigma_{t1} = \frac{-\,2p\,(r_2)^2}{(r_2)^2 - (r_1)^2} = \frac{-\,2 \times 8\,(80)^2}{(80)^2 - (60)^2} = -36.6\ \text{N/mm}^2$$

$= 36.6$ MPa (compressive)

Tangential stress at the outer surface of the inner tube,

$$\sigma_{t2} = \frac{-\,p\,[(r_2)^2 + (r_1)^2]}{(r_2)^2 - (r_1)^2} = \frac{-\,8\,[(80)^2 + (60)^2]}{(80)^2 - (60)^2} = -28.6\ \text{N/mm}^2$$

$= 28.6$ MPa (compressive)

Tangential stress at the inner surface of the outer tube,

$$\sigma_{t3} = \frac{p\,[(r_3)^2 + (r_2)^2]}{(r_3)^2 - (r_2)^2} = \frac{8\,[(100)^2 + (80)^2]}{(100)^2 - (80)^2} = 36.4\ \text{N/mm}^2$$

$= 36.4$ MPa (tensile)

and tangential stress at the outer surface of the outer tube,

$$\sigma_{t4} = \frac{2p\,(r_2)^2}{(r_3)^2 - (r_2)^2} = \frac{2 \times 8\,(80)^2}{(100)^2 - (80)^2} = 28.4\ \text{N/mm}^2$$

$= 28.4$ MPa (tensile)

Now let us find out the stresses induced in the assembly due to internal fluid pressure (p_i).

We know that the tangential stress at the inner surface of the inner tube,

$$\sigma_{t5} = \frac{p_i\,[(r_3)^2 + (r_1)^2]}{(r_3)^2 - (r_1)^2} = \frac{60\,[(100)^2 + (60)^2]}{(100)^2 - (60)^2} = 127.5\ \text{N/mm}^2$$

$= 127.5$ MPa (tensile)

Top

Tangential stress at the outer surface of the inner tube or inner surface of the outer tube (*i.e.*, mating surface),

$$\sigma_{t6} = \frac{p_i (r_1)^2}{(r_2)^2}\left[\frac{(r_3)^2 + (r_2)^2}{(r_3)^2 - (r_1)^2}\right] = \frac{60\,(60)^2}{(80)^2}\left[\frac{(100)^2 + (80)^2}{(100)^2 - (60)^2}\right] = 86.5 \text{ N/mm}^2$$

$= 86.5$ MPa (tensile)

and tangential stress at the outer surface of the outer tube,

$$\sigma_{t7} = \frac{2\,p_i\,(r_1)^2}{(r_3)^2 - (r_1)^2} = \frac{2 \times 60\,(60)^2}{(100)^2 - (60)^2} = 67.5 \text{ N/mm}^2 = 67.5 \text{ MPa (tensile)}$$

We know that resultant stress at the inner surface of the assembly

$$\sigma_{ti} = \sigma_{t1} + \sigma_{t5} = -36.6 + 127.5 = 90.9 \text{ N/mm}^2 = 90.9 \text{ MPa (tensile)}$$ **Ans.**

Resultant stress at the outer surface of the inner tube

$$= \sigma_{t2} + \sigma_{t6} = -28.6 + 86.5 = 57.9 \text{ N/mm}^2 = 57.9 \text{ MPa (tensile)}$$

Resultant stress at the inner surface of the outer tube

$$= \sigma_{t3} + \sigma_{t6} = 36.4 + 86.5 = 122.9 \text{ N/mm}^2 = 122.9 \text{ MPa (tensile)}$$

∴ Total resultant stress at the mating surface of the assembly,

$$\sigma_{tm} = 57.9 + 122.9 = 180.8 \text{ N/mm}^2 = 180.8 \text{ MPa (tensile)}$$ **Ans.**

and resultant stress at the outer surface of the assembly,

$$\sigma_{to} = \sigma_{t4} + \sigma_{t7} = 28.4 + 67.5 = 95.9 \text{ N/mm}^2 = 95.9 \text{ MPa (tensile)}$$ **Ans.**

7.12 Cylinder Heads and Cover Plates

The heads of cylindrical pressure vessels and the sides of rectangular or square tanks may have flat plates or slightly dished plates. The plates may either be cast integrally with the cylinder walls or fixed by means of bolts, rivets or welds. The design of flat plates forming the heads depend upon the following two factors:

(*a*) Type of connection between the head and the cylindrical wall, (*i.e.* freely supported or rigidly fixed); and

(*b*) Nature of loading (*i.e.* uniformly distributed or concentrated).

Since the stress distribution in the cylinder heads and cover plates are of complex nature, therefore empirical relations based on the work of Grashof and Bach are used in the design of flat plates. Let us consider the following cases:

This 2500-ton hydraulic press is used to forge machine parts a high temperature.

Note : This picture is given as additional information and is not a direct example of the current chapter.

1. ***Circular flat plate with uniformly distributed load.*** The thickness (t_1) of a plate with a diameter (d) supported at the circumference and subjected to a pressure (p) uniformly distributed over the area is given by

$$t_1 = k_1 . d \sqrt{\frac{p}{\sigma_t}}$$

where σ_t = Allowable design stress.

Contents

The coefficient k_1 depends upon the material of the plate and the method of holding the edges. The values of k_1 for the cast iron and mild steel are given in Table 7.2.

2. *Circular flat plate loaded centrally.* The thickness (t_1) of a flat cast iron plate supported freely at the circumference with a diameter (d) and subjected to a load (F) distributed uniformly over an area $\frac{\pi}{4}(d_0)^2$, is given by

$$t_1 = 3\sqrt{\left(1 - \frac{0.67\, d_0}{d}\right)\frac{F}{\sigma_t}}$$

If the plate with the above given type of loading is fixed rigidly around the circumference, then

$$t_1 = 1.65\sqrt{\frac{F}{\sigma_t}\log_e\left(\frac{d}{d_0}\right)}$$

3. *Rectangular flat plate with uniformly distributed load.* The thickness (t_1) of a rectangular plate subjected to a pressure (p) uniformly distributed over the total area is given by

$$t_1 = a.b.k_2\sqrt{\frac{p}{\sigma_t(a^2 + b^2)}}$$

where a = Length of the plate; and

b = Width of the plate.

The values of the coefficient k_2 are given in Table 7.2.

Table 7.2. Values of coefficients k_1, k_2, k_3 and k_4.

Material of the cover plate	*Type of connection*	*Circular plate*	*Rectangular plate*		*Elliptical plate*
		k_1	k_2	k_3	k_4
Cast iron	Freely supported	0.54	0.75	4.3	1.5
	Fixed	0.44	0.62	4.0	1.2
Mild Steel	Freely supported	0.42	0.60	3.45	1.2
	Fixed	0.35	0.49	3.0	0.9

4. *Rectangular flat plate with concentrated load.* The thickness (t_1) of a rectangular plate subjected to a load (F) at the intersection of the diagonals is given by

$$t_1 = k_3\sqrt{\frac{a.b.F}{\sigma_t(a^2 + b^2)}}$$

The values of coefficient k_3 are given in Table 7.2.

5. *Elliptical plate with uniformly distributed load.* The thickness (t_1) of an elliptical plate subjected to a pressure (p) uniformly distributed over the total area, is given by

$$t_1 = a.b.k_4\sqrt{\frac{p}{\sigma_t(a^2 + b^2)}}$$

where a and b = Major and minor axes respectively.

The values of coefficient k_4 are given in Table 7.2.

Top

6. *Dished head with uniformly distributed load*. Let us consider the following cases of dished head:

(*a*) *Riveted or welded dished head*. When the cylinder head has a dished plate, then the thickness of such a plate that is riveted or welded as shown in Fig. 7.12 (*a*), is given by

$$t_1 = \frac{4.16\ p.R}{\sigma_u}$$

where p = Pressure inside the cylinder,

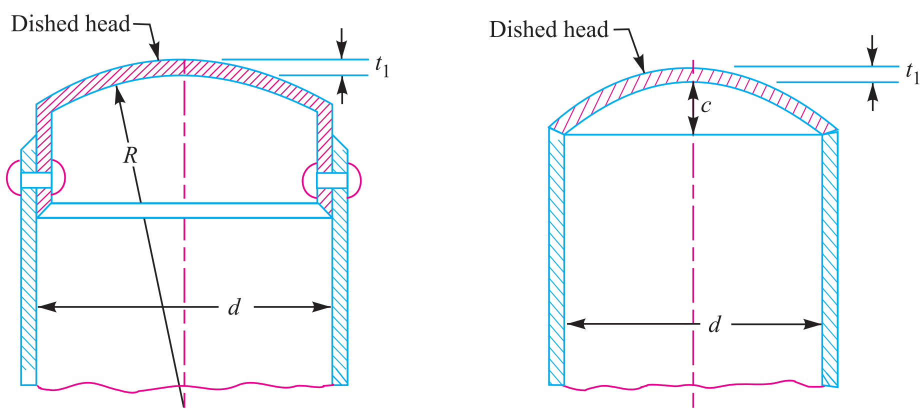
R = Inside radius of curvature of the plate, and

σ_u = Ultimate strength for the material of the plate.

When there is an opening or manhole in the head, then the thickness of the dished plate is given by

$$t_1 = \frac{4.8\ p.R}{\sigma_u}$$

It may be noted that the inside radius of curvature of the dished plate (R) should not be greater than the inside diameter of the cylinder (d).



(*a*) Riveted or welded dished head. (*b*) Integral or welded dished head.

Fig. 7.12. Dished plate with uniformly distributed load.

(*b*) *Integral or welded dished head*. When the dished plate is fixed integrally or welded to the cylinder as shown in Fig. 7.12 (*b*), then the thickness of the dished plate is given by

$$t_1 = \frac{p\,(d^2 + 4c^2)}{16\ \sigma_t \times c}$$

where c = Camber or radius of the dished plate.

Mostly the cylindrical shells are provided with hemispherical heads. Thus for hemispherical heads, $c = \frac{d}{2}$. Substituting the value of c in the above expression, we find that the thickness of the hemispherical head (fixed integrally or welded),

$$t_1 = \frac{p\left(d^2 + 4 \times \frac{d^2}{4}\right)}{16\ \sigma_t \times \frac{d}{2}} = \frac{p.d}{4\ \sigma_t} \quad \text{...(Same as for thin spherical shells)}$$

Contents

7. ***Unstayed flat plate with uniformly distributed load.*** The minimum thickness (t_1) of an unstayed steel flat head or cover plate is given by

$$t_1 = d\sqrt{\frac{k.p}{\sigma_t}}$$

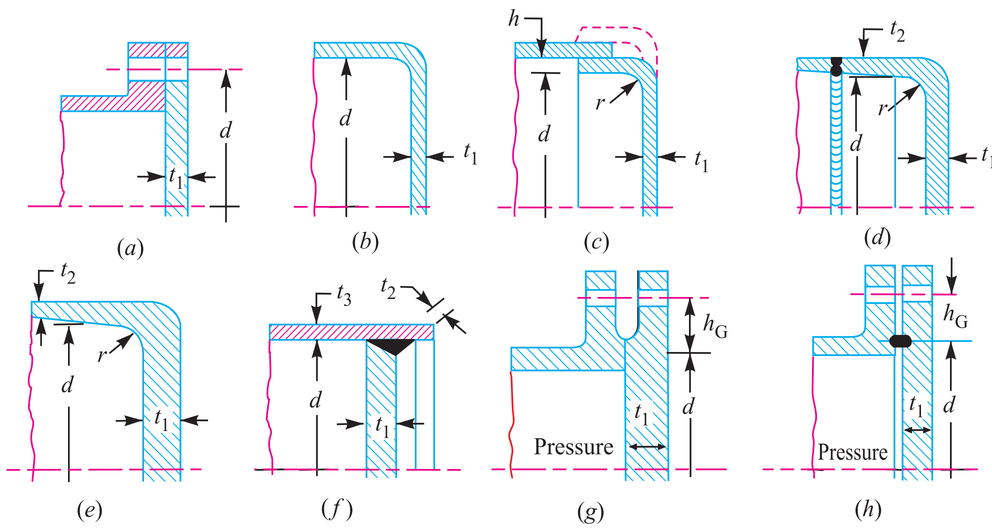


Fig. 7.13. Types of unstayed flat head and covers.

The following table shows the value of the empirical coefficient (k) for the various types of plate (or head) connection as shown in Fig. 7.13.

Table 7.3. Values of an empirical coefficient (k).

S.No.	*Particulars of plate connection*	*Value of 'k'*
1.	Plate riveted or bolted rigidly to the shell flange, as shown in Fig. 7.13 (*a*).	0.162
2.	Integral flat head as shown in Fig. 7.13 (*b*), $d \leq 600$ mm, $t_1 \geq 0.05\ d$.	0.162
3.	Flanged plate attached to the shell by a lap joint as shown in Fig. 7.13 (*c*), $r \geq 3t_1$.	0.30
4.	Plate butt welded as shown in Fig. 7.13 (*d*), $r \geq 3\ t_2$	0.25
5.	Integral forged plate as shown in Fig. 7.13 (*e*), $r \geq 3\ t_2$	0.25
6.	Plate fusion welded with fillet weld as shown in Fig. 7.13 (*f*), $t_2 \geq 1.25\ t_3$.	0.50
7.	Bolts tend to dish the plate as shown in Fig. 7.13 (*g*) and (*h*).	$0.3 + \frac{1.04\ W.h_G}{H.d}$, W = Total bolt load, and H = Total load on area bounded by the outside diameter of the gasket.

Top

Contents

Example 7.15. *A cast iron cylinder of inside diameter 160 mm is subjected to a pressure of 15 N/mm². The permissible working stress for the cast iron may be taken as 25 MPa. If the cylinder is closed by a flat head cast integral with the cylinder walls, find the thickness of the cylinder wall and the flat head.*

Solution. Given : d_i = 160 mm or r_i = 80 mm ; p = 15 N/mm² ; σ_t = 25 MPa = 25 N/mm²

Thickness of the cylinder wall

We know that the thickness of the cylinder wall,

$$t = r_i\left[\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1\right] = 80\left[\sqrt{\frac{25+15}{25-15}} - 1\right] = 80 \text{ mm} \quad \textbf{Ans.}$$

Thickness of the flat head

Since the head is cast integral with the cylinder walls, therefore from Table 7.2, we find that k_1 = 0.44.

∴ Thickness of the flat head,

$$t_1 = k_1.d\sqrt{\frac{p}{\sigma_t}} = 0.44 \times 160\sqrt{\frac{15}{25}} = 54.5 \text{ say } 60 \text{ mm } \textbf{Ans.}$$

Example 7.16. *The steam chest of a steam engine is covered by a flat rectangular plate of size 240 mm by 380 mm. The plate is made of cast iron and is subjected to a steam pressure of 1.2 N/mm². If the plate is assumed to be uniformly loaded and freely supported at the edges, find the thickness of plate for an allowable stress of 35 N/mm².*

Steam engine.

Solution. Given: b = 240 m ; a = 380 mm ; p = 1.2 N/mm² ; σ_t = 35 N/mm²

From Table 7.2, we find that for a rectangular plate freely supported, the coefficient k_2 = 0.75.

We know that the thickness of a rectangular plate,

$$t_1 = a.b.k_2\sqrt{\frac{p}{\sigma_t(a^2+b^2)}} = 380 \times 240 \times 0.75\sqrt{\frac{1.2}{35[(380)^2+(240)^2]}}$$

$$= 68\,400 \times 0.412 \times 10^{-3} = 28.2 \text{ say } 30 \text{ mm } \textbf{Ans.}$$

Example 7.17. *Determine the wall thickness and the head thickness required for a 500 mm fusion-welded steel drum that is to contain ammonia at 6 N/mm² pressure. The radius of curvature of the head is to be 450 mm.*

Solution. Given: d = 500 mm ; p = 6 N/mm² ; R = 450 mm

Wall thickness for a steel drum

For the chemical pressure vessels, steel Fe 360 is used. The ultimate tensile strength (σ_u) of the steel is 360 N/mm². Assuming a factor of safety ($F.S.$) as 6, the allowable tensile strength,

$$\sigma_t = \frac{\sigma_u}{F.S.} = \frac{360}{6} = 60 \text{ N/mm}^2$$

We know that the wall thickness,

$$t = \frac{p.d}{2\sigma_t} = \frac{6 \times 500}{2 \times 60} = 25 \text{ mm } \textbf{Ans.}$$

Top

Head thickness for a steel drum

We know that the head thickness,

$$t_1 = \frac{4.16\ p.R}{\sigma_u} = \frac{4.16 \times 6 \times 450}{360} = 31.2 \text{ say } 32 \text{ mm } \textbf{Ans.}$$

Example 7.18. *A pressure vessel consists of a cylinder of 1 metre inside diameter and is closed by hemispherical ends. The pressure intensity of the fluid inside the vessel is not to exceed 2 N/mm². The material of the vessel is steel, whose ultimate strength in tension is 420 MPa. Calculate the required wall thickness of the cylinder and the thickness of the hemispherical ends, considering a factor of safety of 6. Neglect localised effects at the junction of the cylinder and the hemisphere.*

Solution. Given: d = 1 m = 1000 mm ; p = 2 N/mm^2 ; σ_u = 420 MPa = 420 N/mm^2 ; $F.S.$ = 6

We know that allowable tensile stress,

$$\sigma_t = \frac{\sigma_u}{F.S.} = \frac{420}{6} = 70 \text{ N/mm}^2$$

Wall thickness of the cylinder

We know that wall thickness of the cylinder,

$$t = \frac{p.d}{2\,\sigma_t} = \frac{2 \times 1000}{2 \times 70} = 14.3 \text{ say } 15 \text{ mm } \textbf{Ans.}$$

Thickness of hemispherical ends

We know that the thickness of hemispherical ends,

$$t_1 = \frac{p.d}{4\,\sigma_t} = \frac{2 \times 1000}{4 \times 70} = 7.15 \text{ say } 8 \text{ mm } \textbf{Ans.}$$

Example 7.19. *A cast steel cylinder of 350 mm inside diameter is to contain liquid at a pressure of 13.5 N/mm². It is closed at both ends by flat cover plates which are made of alloy steel and are attached by bolts.*

1. *Determine the wall thickness of the cylinder if the maximum hoop stress in the material is limited to 55 MPa.*
2. *Calculate the minimum thickness necessary of the cover plates if the working stress is not to exceed 65 MPa.*

Steel drums.

Solution. Given : d_i = 350 mm or r_i = 175 mm ; p = 13.5 N/mm^2 ; σ_t = 55 MPa = 55 N/mm^2 ; σ_{t1} = 65 MPa = 65 N/mm^2

1. Wall thickness of the cylinder

We know that the wall thickness of the cylinder,

$$t = r_i\left[\sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1\right] = 175\left[\sqrt{\frac{55 + 13.5}{55 - 13.5}} - 1\right] = 49.8 \text{ say } 50 \text{ mm } \textbf{Ans.}$$

2. Minimum thickness of the cover plates

From Table 7.3, we find that for a flat cover plate bolted to the shell flange, the value of coefficient k = 0.162. Therefore, minimum thickness of the cover plates

$$t_1 = d_i\sqrt{\frac{k.p}{\sigma_{t1}}} = 350\sqrt{\frac{0.162 \times 13.5}{64}} = 64.2 \text{ say } 65 \text{ mm } \textbf{Ans.}$$

EXERCISES

1. A steel cylinder of 1 metre diameter is carrying a fluid under a pressure of 10 N/mm^2. Calculate the necessary wall thickness, if the tensile stress is not to exceed 100 MPa. **[Ans. 50 mm]**
2. A steam boiler, 1.2 metre in diameter, generates steam at a gauge pressure of 0.7 N/mm^2. Assuming the efficiency of the riveted joints as 75% , find the thickness of the shell. Given that ultimate tensile stress = 385 MPa and factor of safety = 5. **[Ans. 7.3 mm]**
3. Find the thickness of a cast iron cylinder 250 mm in diameter to carry a pressure of 0.7 N/mm^2. Take maximum tensile stress for cast iron as 14 MPa. **[Ans. 6.25 mm]**
4. A pressure vessel has an internal diameter of 1 m and is to be subjected to an internal pressure of 2.75 N/mm^2 above the atmospheric pressure. Considering it as a thin cylinder and assuming the efficiency of its riveted joint to be 79%, calculate the plate thickness if the tensile stress in the material is not to exceed 88 MPa. **[Ans. 20 mm]**
5. A spherical shell of 800 mm diameter is subjected to an internal pressure of 2 N/mm^2. Find the thickness required for the shell if the safe stress is not to exceed 100 MPa. **[Ans. 4 mm]**
6. A bronze spherical shell of thickness 15 mm is installed in a chemical plant. The shell is subjected to an internal pressure of 1 N/mm^2. Find the diameter of the shell, if the permissible stress for the bronze is 55 MPa. The efficiency may be taken as 80%. **[Ans. 2.64 m]**
7. The pressure within the cylinder of a hydraulic press is 8.4 N/mm^2. The inside diameter of the cylinder is 25.4 mm. Determine the thickness of the cylinder wall, if the allowable tensile stress is 17.5 MPa. **[Ans. 8.7 mm]**
8. A thick cylindrical shell of internal diameter 150 mm has to withstand an internal fluid pressure of 50 N/mm^2. Determine its thickness so that the maximum stress in the section does not exceed 150 MPa. **[Ans. 31 mm]**
9. A steel tank for shipping gas is to have an inside diameter of 30 mm and a length of 1.2 metres. The gas pressure is 15 N/mm^2. The permissible stress is to be 57.5 MPa. **[Ans. 4.5 mm]**
10. The ram of a hydraulic press 200 mm internal diameter is subjected to an internal pressure of 10 N/mm^2. If the maximum stress in the material of the wall is not to exceed 28 MPa, find the external diameter. **[Ans. 265 mm]**
11. The maximum force exerted by a small hydraulic press is 500 kN. The working pressure of the fluid is 20 N/mm^2. Determine the diameter of the plunger, operating the table. Also suggest the suitable thickness for the cast steel cylinder in which the plunger operates, if the permissible stress for cast steel is 100 MPa. **[Ans. 180 mm ; 20 mm]**
12. Find the thickness of the flat end cover plates for a 1 N/mm^2 boiler that has a diameter of 600 mm. The limiting tensile stress in the boiler shell is 40 MPa. **[Ans. 38 mm]**

This vessel holds oil at high pressure.

Contents

QUESTIONS

1. What is the pressure vessel ?
2. Make out a systematic classification of pressure vessels and discuss the role of statutory regulations.
3. How do you distinguish between a thick and thin cylinder?
4. What are the important points to be considered while designing a pressure vessel ?
5. Distinguish between circumferential stress and longitudinal stress in a cylindrical shell, when subjected to an internal pressure.
6. Show that in case of a thin cylindrical shell subjected to an internal fluid pressure, the tendency to burst lengthwise is twice as great as at a transverse section.
7. When a thin cylinder is subjected to an internal pressure p, the tangential stress should be the criterion for determining the cylinder wall thickness. Explain.
8. Derive a formula for the thickness of a thin spherical tank subjected to an internal fluid pressure.
9. Compare the stress distribution in a thin and thick walled pressure vessels.
10. When the wall thickness of a pressure vessel is relatively large, the usual assumptions valid in thin cylinders do not hold good for its analysis. Enumerate the important violations. List any two theories suggested for the analysis of thick cylinders.
11. Discuss the design procedure for pressure vessels subjected to higher external pressure.
12. Explain the various types of ends used for pressure vessel giving practical applications of each.

OBJECTIVE TYPE QUESTIONS

1. A pressure vessel is said to be a thin cylindrical shell, if the ratio of the wall thickness of the shell to its diameter is

   (*a*) equal to 1/10 (*b*) less than 1/10
   (*c*) more than 1/10 (*d*) none of these

2. In case of pressure vessels having open ends, the fluid pressure induces

   (*a*) longitudinal stress (*b*) circumferential stress
   (*c*) shear stress (*d*) none of these

3. The longitudinal stress is ...... of the circumferential stress.

   (*a*) one-half (*b*) two-third
   (*c*) three-fourth

4. The design of the pressure vessel is based on

   (*a*) longitudinal stress (*b*) hoop stress
   (*c*) longitudinal and hoop stress (*d*) none of these

5. A thin spherical shell of internal diameter d is subjected to an internal pressure p. If σ_t is the tensile stress for the shell material, then thickness of the shell (t) is equal to

   (*a*) $\frac{p.d}{\sigma_t}$ (*b*) $\frac{p.d}{2\,\sigma_t}$
   (*c*) $\frac{p.d}{3\,\sigma_t}$ (*d*) $\frac{p.d}{4\,\sigma_t}$

Top

6. In case of thick cylinders, the tangential stress across the thickness of cylinder is
   (*a*) maximum at the outer surface and minimum at the inner surface
   (*b*) maximum at the inner surface and minimum at the outer surface
   (*c*) maximum at the inner surface and zero at the outer surface
   (*d*) maximum at the outer surface and zero at the inner surface
7. According to Lame's equation, the thickness of a cylinder is equal to

   (*a*) $r_i\left[\sqrt{\dfrac{\sigma_t + (1-2\mu)\,p}{\sigma_t - (1-2\mu)\,p}} - 1\right]$ (*b*) $r_i\left[\sqrt{\dfrac{\sigma_t + (1-\mu)\,p}{\sigma_t - (1-\mu)\,p}} - 1\right]$

   (*c*) $r_i\left[\sqrt{\dfrac{\sigma_t + p}{\sigma_t - p}} - 1\right]$ (*d*) $r_i\left[\sqrt{\dfrac{\sigma_t}{\sigma_t - 2p}} - 1\right]$

   where
   r_i = Internal radius of the cylinder,
   σ_t = Allowable tensile stress,
   p = Internal fluid pressure, and
   μ = Poisson's ratio.
8. In a thick cylindrical shell, the maximum radial stress at the outer surfaces of the shell is
   (*a*) zero (*b*) p
   (*c*) $-p$ (*d*) $2p$
9. For high pressure oil and gas cylinders, the thickness of the cylinder is determined by
   (*a*) Lame's equation (*b*) Clavarino's equation
   (*c*) Barlow's equation (*d*) Birnie's equation
10. The thickness of a dished head that is riveted or welded to the cylindrical wall is

   (*a*) $\dfrac{4.16\ p.R}{\sigma_u}$ (*b*) $\dfrac{5.36\ p.R}{\sigma_u}$

   (*c*) $\dfrac{6.72\ p.R}{\sigma_u}$ (*d*) $\dfrac{8.33\ p.R}{\sigma_u}$

   where
   p = Internal pressure,
   R = Inside radius of curvature of the dished plate, and
   σ_u = Ultimate strength for the material of the plate.

ANSWERS

1. (*b*)	**2.** (*b*)	**3.** (*a*)	**4.** (*b*)	**5.** (*b*)
6. (*b*)	**7.** (*c*)	**8.** (*a*)	**9.** (*c*)	**10.** (*a*)